Engineering Fracture Mechanics 119 (2014) 53–65

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Simulation of time-dependent crack growth in brittle rocks under constant loading conditions

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ARTICLE INFO

Article history:
Received 26 June 2013
Received in revised form 4 December 2013
Accepted 12 February 2014

Keywords:
Lifetime
Subcritical crack growth
Charles equation
Wing crack
Numerical simulation
Rock mechanics

ABSTRACT

Based on the theory of subcritical crack growth, linear elastic fracture mechanics (LEFM), and Charles equation, a lifetime prediction scheme has been developed for rock specimens containing initial microcracks under constant loadings. Numerical simulations were performed utilizing the developed modeling scheme. Lifetimes were obtained through numerical calculation; the damage process and macroscopic fracture pattern of the models were studied. Typical fracture pattern, like tensile cracks and shear bands, were observed. A few preliminary studies were also performed to compare the results with in situ observations. Conclusions were drawn and possible improvements to future research work are proposed.

1. Introduction

Experiments and theoretical research on the phenomenon of fracture and damage started centuries ago. But no quantitative results were obtained until the great work of Griffith [1], which had lead to the onset of modern fracture mechanics. Griffith analyzed the stresses of a cracked plate, which was studied before by Inglis [2], and develop a theory for crack initiation and propagation. Orowan [3] modified Griffith's theory by including the influence of plasticity into the energy balance. Irwin [4] introduced the parameter later known as the stress intensity factor K. After the fundamentals of linear elastic fracture mechanics were well established around 1960 [5], researchers had focused more on the influence of materials' plasticity on the fracture analysis. One trend in recent fracture and damage mechanical research is related to time-dependent studies (life time prediction or time to failure prediction) for rocks (e.g. Kemeny [6,7,8], Mishnaevsky [9], Shao et al. [10], Li et al. [11], Rinne [12], Konietzky et al. [13]).

Based on the linear elastic fracture mechanical theory (LEFM), numerical simulations of time-dependent fracture propagation on Westerly Granite have been performed by Konietzky et al. [13]. The innovations of their research work include the simulation of specific initial cracks with certain distributions at the micro-scale, and the simulation of sub-critical and critical fracture growth to describe the time-dependent damage process until failure. It is assumed, that the damage and finally the failure of a rock specimen is the result of the partially parallel growth and coalescence of many initially existing microcracks at the grain size level rather than the growth of one or only a few single cracks. Other studies, like the experimental investigations on dynamic fracture performed by Ravi-Chandar and Knauss [14,15], also implied this concept. As a further improvement of the work of Konietzky et al. [13], this study includes the influence of orientation distribution of the initial

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http://dx.doi.org/10.1016/j.engfracmech.2014.02.008
0013-7944/

microcracks on the crack growth, and an adopted wing crack propagation scheme to describe the growth of initial microcracks.

2. Numerical simulation idea

2.1. Theoretical basis

Irwin [4] introduced stress intensity factor to describe the stress distribution and displacements near the crack tip of brittle materials. A fracture criterion based on the stress intensity factor can be described as follows: failure of the material occurs when the stress intensity factor reaches the critical value, the so-called fracture toughness K_C. It was assumed by classical LEFM that the crack will propagate ultrasonically when the stress intensity factor K reaches the fracture toughness K_C. Otherwise the crack will remain stable. However, the phenomenon known as subcritical crack growth was observed where crack propagates at certain low velocity when stress intensity factor is lower than the critical value K_C (Wiederhorn [16], Atkinson [17][18], Dill et al. [19], Ritter et al. [20], Nara and Kaneko [21]). The mechanism of subcritical crack growth has been studied and explained by means of different theories (Fuller and Thomson [22]; Rice [23]; Wiederhorn and Bloz [24]; Lawn [25]; Brown [26]). Charles equation is most utilized to describe the subcritical crack growth (stress corrosion) within the reaction rate theory. Charles [27] studied the delayed failure of glass and the expression of crack growth velocity for subcritical crack growth was obtained:

$$v = v_0 K^n \exp\left(\frac{-u}{RT}\right) \tag{1}$$

where v_0 is a material constant; K is the stress intensity factor; n is the stress corrosion index; u is the activation energy; T is the absolute temperature and R is the gas constant (Boltzmann constant). With the same rock type and in the constant temperature, the component $v_0 \exp(-u/(RT))$ in Eq. (1) is a constant, thus can be substituted by a rock-specific parameter C (fracture growth constant C in Table 1). In this case, Charles equation is expressed as (Konietzky et al. [13]):

$$v = CK^n \tag{2}$$

Based on the lab data of Westerly Granite [28], the values for C and n were deduced: $C = 8.8552 \times 10^{-214}$ and $n = 33.7234$. In this study, Eq. (2) is utilized to obtain the velocity at which the microcrack propagates subcritical. When the crack has come to the critical state ($K \geqslant K_C$), Eq. (2) is no longer applicable and the crack propagation is considered reached ultrasonic speed.

2.2. Wing crack propagation scheme

It is observed that cracks under load typically propagate forming wing cracks in the direction of the maximum circumferential stress. This propagation pattern is adopted in this study based on the simulation idea proposed by Baud et al. [29], where the curved wing crack is simplified to a straight one by connecting the original main crack tip to the wing crack tip. This innovative model is a simplification to the exact local stress intensity factor derivation for the wing crack system proposed by Nemat-Nasser and Horii [30] and Horii and Nemat-Nasser [31].

Consider an initial straight crack OO' under load as shown in Fig. 1. After one calculation step (time step) the real crack propagation path is represented by Curve OA (Fig. 1(a)). Replacing Curve OA with straight red line OA, local stress intensity factor K_A is then calculated using the length OA and its orientation. This K_A value is used to calculate crack propagation velocity through Charles equation (Eq. (1)) for the next time step. In the next time step, wing crack continues propagating in the direction of the maximum circumferential stress based on the stress field of this step. Assuming in the current time step, the crack tip has reached Point B, and real crack path forms Curve AB, and Point B becomes the current wing crack tip. Then

Table 1
Material parameters (Konietzky et al. [13]).

Young's modulus	73.8 GPa
Poisson's ratio	0.22
Bulk modulus	43.9 GPa
Shear modulus	30.2 GPa
Density	2700 kg/m^3
Initial cohesion	45 MPa
Initial friction angle	30°
Initial tensile strength	15 MPa
Residual cohesion	0
Residual friction angle	30°
Residual tensile strength	0
Stress corrosion index n	33.7234
Fracture toughness K_{IC}	1.79 MPa m$^{1/2}$
Fracture growth constant C	8.8552×10^{-214} m s^{-1}·(Pa m$^{1/2}$)$^{-n}$