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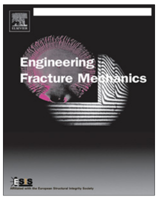

A two-parameter fracture mechanics model for fatigue crack growth in brittle materials

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ABSTRACT

Nonstandard samples were used to conduct fatigue experiments on brittle material (PMMA) to study the effect of local state of stress at crack tip on fatigue crack propagation rate (FCPR). Results indicated that FCPR cannot be generally characterized by cyclic stress intensity factor range alone and that a second parameter representing the influence of *K*-dominance zone size and nonsingular stress field is needed. New phenomenological model based on the two-parameter fracture mechanics is proposed for FCPR prediction. Model allows accurate transferability of laboratory data to life prediction in service situations of defects in engineering components under fatigue conditions.

1. Introduction

The importance of fatigue, the special behavior pattern exhibited by materials in response to cyclic loading, is evident: repeated loadings result in crack nucleation, further fatigue damage (first micro then macro-crack growth) during limited exposures and eventual failure (fracture) during continued exposures. Testing and characterization of fatigue crack growth are used extensively to predict the rate at which subcritical cracks grow in specimen material due to fatigue loading. In engineering applications fatigue crack propagation rate data is essential for reliable life prediction and for calculating in-service nondestructive inspection intervals for safe maintenance of damage tolerant structures.

In the past many investigators have carried out research of subcritical crack propagation in metals and polymeric materials basing on linear elastic fracture mechanics (LEFM). This approach was originally introduced in 1961 by Paris and others [1] who demonstrated that in high cycle fatigue region (i.e. under small scale yielding conditions) a correlation between crack growth rate, da/dN, and the cyclic stress intensity parameter, ΔK, exists. It was argued that ΔK characterizes the magnitude of the fatigue stresses in the crack-tip region and, hence, it should characterize the crack growth rate. Within the frame of this approach a vast variety of models for crack growth prediction that account for different loading conditions (mean values of stress intensity, loading frequencies, load ratios, environment) were formulated, see Refs. [2,3] for information. In the simplest form the data for midrange fatigue crack growth rate values is represented by Eq. (1), commonly known as the Paris law:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = A_P \cdot (\Delta K)^{m_P} \quad (1)$$

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Nomenclature

a	crack length
a_0	initial crack length (from machined slit)
A_P	constant of the Paris–Erdogan equation
A_1, A_2, A_3	first three terms in Williams series expansion
A_3^{max}, A_3^{min}	maximum and minimum values of the third term in Williams expansion in a load cycle
ΔA_3	the cyclic range of the third term in Williams expansion
ΔC	slope of the near crack tip cyclic non-singular opening stress range
ΔC^{APP}	apparent slope of the near crack tip cyclic non-singular opening stress range
$\mathrm{d}a/\mathrm{d}N$	fatigue crack propagation rate
h	characteristic dimension: M(T) sample half-gage length; DCB sample height
K	stress intensity factor (mode I)
K^{max}, K^{min}	maximum and minimum values of stress intensity factor in a load cycle
ΔK	cyclic stress intensity factor range
ΔK^{APP}	apparent stress intensity factor range
ΔK^{REFF}	cyclic stress intensity range corresponding to $\Delta A_3 = 0$
m_P	power exponent of the Paris-Erdogan equation
r, θ	crack tip polar coordinates
R	minimum/maximum cyclic load ratio (stress ratio)
t	M(T) sample thickness (PMMA material thickness)
T	T-stress
w	DCB sample width (PMMA material thickness)
W	M(T) sample width
x, y	crack tip Cartesian coordinates
x_0	finite distance ahead of the crack tip used to correlate the apparent cyclic opening stress range and FCPR
Λ	degree of K-dominance
σ_{ij}	stress tensor in the neighborhood of a planar crack
$\sigma_{\theta\theta}$	tangential (hoop) stress component
σ_y	opening stress component
$\sigma_y^{max}, \sigma_y^{min}$	opening stress distributions in the vicinity of a crack tip under the maximum and minimum loads during one cycle
$\Delta\sigma_y$	cyclic opening stress range
FCPR	fatigue crack propagation rate
LEFM	linear elastic fracture mechanics
M(T)	middle tension sample
DCB	double-cantilever sample

where A_P and m_P are experimentally determined constants. Fatigue crack growth curves defined by (1) are typically obtained on standard laboratory specimens with through-thickness cracks and subsequently considered as characteristic material properties [4]. This fact enables the transferability of the laboratory measurements to design and evaluation of the actual flawed engineering structural components.

The application of conventional assumption of single-valued relationship between $\mathrm{d}a/\mathrm{d}N$ and ΔK to assess the integrity of cracked bodies relies on the notion that a single fracture mechanics parameter, ΔK, uniquely characterizes the response of material to cyclic loading, i.e. the variation of the stress field in the vicinity of a crack tip. Meanwhile it is well recognized by now that in many situations the use of stress intensity alone leads to significant discrepancies from reading the results of fracture and fatigue tests of the same material. It was noticed [5–7] that variable shape and size of laboratory samples may produce essentially different local states of stress at the crack fronts when the stress intensity alone can no longer accurately represent the crack driving force. This phenomenon in literature is usually referred to as constraining effect on the crack and had attracted a great deal of concentrated interest in early 1990s to evaluate various constraint parameters and quantify the influence of constraint on fracture and fatigue crack growth [8,9]. The level of constraint depends upon the crack configuration and crack location relative to external boundaries, the material thickness, the type and magnitude of applied loading, and the material stress–strain properties. Once the level of constraint is quantified, it can further be considered by means of engineering models in conjunction with stress intensity to determine their combined critical values as crack-driving parameters within the scope of two-parameter fracture mechanics approach to describe fracture and fatigue behavior of cracked bodies.

The characterization of constraint has been expressed in terms of the next term(s) in the series expansion representing the elastic or elastic–plastic crack tip stress fields. For ductile materials the influence of T-stress (the first non-singular term in the series expansion of the stress parallel to the crack plane) on the crack-tip plastic zone size and on the fracture and