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Competition of crack and debonding at the interface of a circular rigid inclusion under uniform loading

Norio Hasebe [a,*], Yasumiki Yamamoto [b]

[a] Department of Civil Engineering, Nagoya Institute of Technology, Gokiso-cho, Showa-ku, Nagoya 466, Japan
[b] Metropolitan Express Way Public Cooperation, Kasumigaseki 1-4-1, Chiyoda-ku, Tokyo 100, Japan

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ABSTRACT

Competition of a crack and a debonding at the interface of a circular rigid inclusion in an infinite elastic body is analyzed under uniaxial loading in the *x* and *y* directions, respectively, and under biaxial uniform loading. It is investigated how the debonding develops along the interface of the inclusion from the initial debonding and where the debonding stops and a crack occurs from the tip of debonding. Particularly when there are both possibilities of the debonding development and of the crack occurrence from the tip of the debonding, it can be decided which phenomenon actually occurs. The angles at which the debonding develops and the crack occurs are determined. As the criterion for debonding development and crack occurrence at the debonding tip, strain energy release rates are used. Moreover, the restricting condition is that the normal stress at the interface ahead of the debonding tip is positive and the Mode I stress intensity factor just after crack occurrence is positive. As the loading, the constant load and the gradually increasing load from zero are considered. The stress analysis is carried out as a mixed boundary value problem of plane elasticity. As the stress analysis, the rational mapping function of a sum of fractional expressions and complex stress functions are used and closed form stress functions are derived.

1. Introduction

It is well known that the fracture of the material containing inclusion occurs as debonding and crack due to the local stress concentration at the interface of the inclusion. Generally the rigidities of a base material and inclusion are different. Consequently, stress concentration arises at the interface, and locations where the fracture is most apt to occur exist. According to the results of the experimental observation of materials, in which the rigidity of inclusion is larger than that of base material and the adhesion of the base material with the inclusion is weak such as concrete and high strength steel, it was reported that first, debonding occurs at the interface of inclusion and develops. It is also found that the debonding develops from initial defects at the interface. And it has been investigated that at the tip of debonding which is the singular point of stress, large stress concentration occurs, a crack arises, and thus fracture advances further. Thereupon, when the fracture originating at the inclusion is investigated, it is necessary to consider the problem, in which debonding and crack are coupled.

* Corresponding author. Tel.: +81 528765015.
E-mail address: hasebe@tea.ocn.ne.jp (N. Hasebe).

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Nomenclature

a	radius of a circular rigid inclusion
b	crack length
θ	initial debonding angle (initial interfacial crack angle)
p	uniform load in the x direction
q	uniform load in the y direction
$\omega(\zeta)$	rational mapping function
$\phi(\zeta)$	stress function
$\psi(\zeta)$	stress function
$\chi(\zeta)$	Plemelj function
α and β	the coordinates of the juncture A and B
κ	function of Poisson's ratio ν, $\kappa = 3 - 4\nu$ (plane strain state) and $\kappa = (3 - \nu)/(1 + \nu)$ (generalized plane stress state)
ν	Poisson's ratio
K_I	Mode I stress intensity factor
K_{II}	Mode II stress intensity factor
F_I	non-dimensional Mode I stress intensity factor
F_{II}	non-dimensional Mode II stress intensity factor
$\lvert\beta_0\rvert$	stress intensity of debonding
G	shear modulus
G_c	strain energy release rate of crack occurrence
G_d	strain energy release rate of debonding development
G_{c0}	fracture toughness value of crack occurrence
G_{d0}	fracture toughness value of debonding development
F_d	non-dimensional stress intensity of debonding
σ_r	normal stress near the debonding tip on the interface
$\tau_{r\theta}$	tangential stress near the debonding tip on the interface

In the present paper, a competition problem between a debonding development and a crack occurrence is solved. At the interface of a circular rigid inclusion existing in an infinite elastic body shown in Fig. 1, models of the development of debonding with arbitrary initial angle are analyzed under uniaxial uniform loading in the x and y directions, respectively, and biaxial uniform loading. When the initial debonding does not exist at the first stage, i.e., the inclusion is perfectly adhered to the base material, the debonding must occur at point D in case of the uniform loading in the x direction, because the maximum normal stress occurs at point D. In this case, it is considered that the initial debonding angle is zero. Under the applied load, the conditions under which debonding develops and under which a crack arises at a certain size of debonding are investigated. Particularly when there are both possibilities of debonding development and of crack occurrence from the tip of the debonding, it can be decided which phenomenon actually occurs. The strain energy release rate is used as the fracture criteria of crack initiation and debonding development. It is considered that as the restricting conditions for crack occurrence into the base material, mode I stress intensity factor just after crack occurrence is positive, and for debonding development, the normal stress at the interface ahead of the debonding tip is positive. Moreover, not only the constant load, but also the increasing load from zero are considered. Also the magnitude of the load at which a debonding develops or a crack arises can

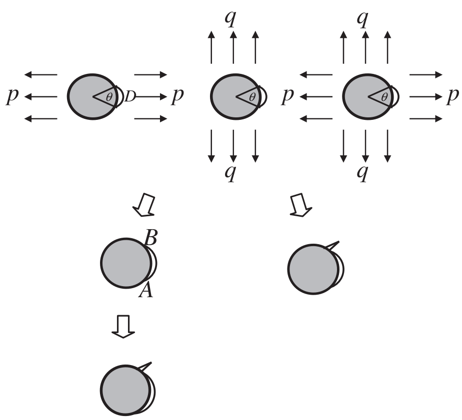

Fig. 1. Competition of a crack occurrence and debonding development at the interface of a circular rigid inclusion under applied load.