Engineering Fracture Mechanics 98 (2013) 365–382

Contents lists available at SciVerse ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

On the use of power series solution method in the crack analysis of brittle materials by indirect boundary element method

Mohammad Fatehi Marji *

Faculty of Mining and Metallurgy, Yazd University, Yazd, Iran

A R T I C L E I N F O

Article history:
Received 19 March 2012
Received in revised form 1 October 2012
Accepted 21 November 2012

Keywords:
Power series solution method (PSS)
Singular integrals
Indirect BEM
Crack analysis

A B S T R A C T

A hybridized boundary element/power series solution method is proposed for evaluating displacements and stresses near the crack tips. In the indirect boundary element method, the special crack tip elements can be used to increase the accuracy of the stress and displacement fields near the crack ends. The special crack tip elements are usually used by assuming the crack mouth opening and crack mouth displacement variations in form of an infinite power series which can end up with the special singular integrals. The aim of this paper is to solve these singular integrals by using an infinite power series. The power series solution method has been used to solve some example problems in fracture mechanics and the corresponding semi-analytical results are compared with those obtained by using the system of partial differential integral equations. This comparison demonstrates the high accuracy of the proposed method.

1. Introduction

A variety of analytical solutions such as the power series method, Fourier method, integral transform method, complex variable method [1–7]; the approximate solution procedures which are based on the variational methods (the methods related to energy theorems) such as the minimum potential energy method and Ritz method [8–11]; the semi-analytical procedures such as the indirect boundary element method which is based on the semi-analytical solution of a system of integral equations and/or a system of partial differential integral equations on the boundary of the physical problems [12–14]; and also the numerical procedures such as the finite difference method (FDM), the finite element method (FEM), the direct boundary element method (BEM) or boundary integral method (BIM), and the dual boundary element method (DBEM) can be used for the solution of various complex problems in elasticity and fracture mechanics [15–26]. Recently, the author and his colleagues have been published some papers about the uses of the higher order displacement discontinuity elements and higher order special crack tip elements in the indirect boundary element method to increase the accuracy of the first and second mode stress intensity factors which are important in the study of rock fracture mechanics [26–29]. The method further developed to solve the kinked and curved crack problems [30,31]. In all of the previous works, a system of partial differential integral equations was solved on the boundary of any boundary value problem (BVP) occur in the field of rock fracture mechanics. The method was also extended to solve the infinite, finite and semi-infinite problems in elasticity and fracture mechanics [27]. In the present study, a hybridized semi-analytical method is proposed which incorporates the higher orders indirect boundary element method and the power series solution method for the crack analysis of the finite, infinite and semi-infinite plane elasticity problems.

* Tel.: +98 9133538868; fax: +98 3518210995.
E-mail address: mfatehi@yazduni.ac.ir

0013-7944/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.engfracmech.2012.11.015

Nomenclature

a	element length (one half)
b	crack length
COD	crack opening displacement (normal displacement discontinuity component D_y)
CSD	crack sliding displacement (shear displacement discontinuity component D_x)
$D_i(\varepsilon)$	displacement discontinuity variation along the element
E	Young's modulus
$f(x,y)$, $g(x,y)$	harmonic functions
$f_C(x,y)$, $g_C(x,y)$	special potential functions for the crack tip elements
$F_{,x}$, $F_{,y}$, $F_{,xy}$, etc.	partial derivatives of the single harmonic function, $F(x,y)$
G	shear modulus
$I_C^1, I_C^2, I_C^3, I_C^4$	special singular integrals
K_I, K_{II}	Modes I and II stress intensity factors
$l = 2(a_1 + a_2 + a_3 + a_4)$	the crack tip element length
$N_{C1}(\varepsilon)$, $N_{C2}(\varepsilon)$, etc.	special shape functions for displacement discontinuities $D_i^1(a), D_i^2(a)$, etc.
P_x, P_y	fictitious stress components
$\kappa = (3 - 4\nu)$	for plane strain condition
$\kappa = (3 - \nu)/(1 + \nu)$	for plane stress condition
ν	Poisson's ratio

2. Development of the basic theory

The displacements and stresses for a line crack in an infinite body along the *x-axis*, can be written in terms of the harmonic functions, $g(x,y)$ and $f(x,y)$, for the two general cases of the indirect boundary element method [12], i.e.

(i) The fictitious stress method (FSM), where the displacements are defined with respect to the fictitious stresses, P_x and P_y as:

$$u_x = \frac{P_x}{2G}[(3-4\nu)F + yF_{,y}] + \frac{P_y}{2G}[-yF_{,x}] \quad \text{and} \quad u_y = \frac{P_x}{2G}[-yF_{,x}] + \frac{P_y}{2G}[(3-4\nu)F - yF_{,y}] \tag{1}$$

and the stresses are:

$$\begin{aligned}
\sigma_{xx} &= P_x[(3-2\nu)F_{,x} + xF_{,xy}] + P_y[2\nu F_{,y} - yF_{,yy}] \\
\sigma_{yy} &= P_x[-(1-2\nu)F_{,x} - xF_{,xy}] + P_y[2(1-\nu)F_{,y} - yF_{,yy}] \\
\sigma_{xy} &= P_x[2(1-\nu)F_{,y} + yF_{,yy}] + P_y[(1-2\nu)F_{,x} - yF_{,xy}]
\end{aligned} \tag{2}$$

where G is shear modulus, ν is the Poisson's ratio, and $F_{,x}$, $F_{,y}$, $F_{,xy}$, etc. are the partial derivatives of the single harmonic function, $F(x,y)$, with respect to x and y.

(ii) The displacement discontinuity method (DDM), where the displacements are:

$$\begin{aligned}
u_x &= [2(1-\nu)f_{,y} - yf_{,xx}] + [-(1-2\nu)g_{,x} - yg_{,xy}] \\
u_y &= [(1-2\nu)f_{,x} - yf_{,xy}] + [2(1-\nu)g_{,y} - yg_{,yy}]
\end{aligned} \tag{3}$$

and the stresses are:

$$\begin{aligned}
\sigma_{xx} &= 2G[2f_{,xy} + yf_{,xyy}] + 2G[g_{,yy} + yg_{,yyy}] \\
\sigma_{yy} &= 2G[-yf_{,xyy}] + 2G[g_{,yy} - yg_{,yyy}] \\
\sigma_{xy} &= 2G[2f_{,yy} + yf_{,yyy}] + 2G[-yg_{,xyy}]
\end{aligned} \tag{4}$$

For the case of using, constant or higher order displacement discontinuity elements, it is possible to write the potential functions, $f(x,y)$ and $g(x,y)$, with respect to a common function, $F(x,y)$, which is the same as the single harmonic function given for the FSM. For example, for the case of using cubic displacement discontinuity elements, the potential functions, $f(x,y)$ and $g(x,y)$, can be defined in terms of the shear and normal displacements, D_x and D_y, as [27]:

$$f(x,y) = \frac{-1}{4\pi(1-\nu)}\sum_{j=1}^{4} D_x^j F_j(I_0, I_1, I_2, I_3), \quad g(x,y) = \frac{-1}{4\pi(1-\nu)}\sum_{j=1}^{4} D_y^j F_j(I_0, I_1, I_2, I_3) \tag{5}$$