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The elastic and plastic constraint parameters for three-dimensional problems

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ABSTRACT

Fracture toughness and the in-plane and out-of-plane constraint effects are studied through experiments and computations for high-strength carbon steel 34XH3MA. The subjects for the studies are single-edge-notched bend specimens under three-point bending and compact specimens under tension. Both types of specimens are of non-standard configuration because the specimen thickness-to-width ratio was varied in the range of 0.1–1.0, and the relative crack length was changed in the range of 0.24–0.64. Characterization of the constraint effects was performed using the non-singular *T*-stresses, the local triaxiality parameter *h*, and the T_Z and T_{ZZ}, factors of the stress-state in a 3D cracked body. For the particular geometries of the specimens considered, the numerical constant of the plastic stress field I_n and the plastic stress intensity factor distributions along the crack front are obtained as a function of the dimensionless crack length and the specimen thickness. It is further demonstrated that the plastic stress intensity factor accounting for the in-plane and out-of-plane constraint effects can be used to characterize the fracture resistance characteristics as a unified single parameter for a variety of specimen geometries.

1. Introduction

Many investigations of the so-called constraint effects have been performed over the last two decades with the aim of improving the approximation of the near-tip crack for both 2D and 3D cases. It is necessary to first keep in mind that the analysis of the constraint effects at fracture is related to the background and the limitation of the HRR-fields [1], [2]. The early works devoted to the constraint problems emphasized that this limitation may be explained by considering the non-singular *T*-stress proposed by Rice [3], which acts parallel to the crack plane. The elastic *T*-stress, the second term of the asymptotic series for the stress field in a linear elastic material, has also been introduced by Eftis and Subramonian [4] to characterize in-plane constraint in mode I and mixed-mode biaxial loading.

Recently, several two parameter models describing elastic–plastic fracture mechanics were introduced to explain some of the restrictions inherent in the one parameter approach based on the *J*-integral. The different sources of changes in the in-plane constraint are associated with the crack size, the geometry of the specimen and the loading conditions and notch effects on the fracture toughness. It was shown that much of the dependence of the fracture toughness on the specimen geometry could be explained by two parameter fracture theories based on the elastic and the elastic–plastic constraint

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Nomenclature

a	crack length
B	thickness
w	width
E	Young's modulus
I_n	governing parameter of elastic–plastic crack-tip stress fields
J	J-integral
h	local triaxiality parameter
K_1	elastic stress intensity factor
K_{max}	maximum value of elastic stress intensity factor
K_P	plastic stress intensity factor
n	strain hardening exponent
P	load
P_{max}	maximum value of load
S_{ij}	deviatoric stress
T	non-singular T-stresses
T_{zz}	T-stress in z-direction
T_z	dimensionless constraint factor
$\bar{\alpha}$	strain hardening coefficient
σ_O	yield stress
σ	nominal stress
σ_{xx}, σ_{yy}, σ_{zz}	stress tensor components
σ_{kk}	hydrostatic stress
v	Poisson's ratio

factors T and Q, respectively [5–14]. Under small- and moderate-scale yielding, both the T and Q approaches are essentially equivalent. These two-parameter descriptions were developed by Li and Wang [5] and Sharma and Aravas [6], the J–T solution (Betegon and Hancock [7]), the J–Q solution (O'Dowd and Shih [8]), the J-A_2 solution (Nikishkov [9]), and higher-order solutions up to five terms were obtained by Yang et al. [10,11]. Anderson [12] and Hebel et al. [13] focused their attention on analyzing the constraint effects in the application to plane problems, including the load bi-axiality influence according to Shlyannikov et al. [14].

In the early 1990s, a number of analytical methods were developed to describe the asymptotic stress and strain fields for plane problems in a power law elastic–plastic material [9–13]. Those methods that already existed are based on the application of the solutions of the higher-order terms using an asymptotic expansion and the separation of variables for the stress function. The first term in the series expansion is the HRR solution or the reference field, after Hutchinson [1] and Rice and Rosengren [2]. The amplitude of the first term was introduced by Rice as the J-integral, which was subsequently represented by Hutchinson [1] as the plastic stress intensity factor. Most of these methods are related to the asymptotic expansion for the pure Mode I case under either plane strain or plane stress conditions for power-law hardening materials and perfectly plastic solids.

It is well known that different traditional approaches, which successfully describe the in-plane constrain, are not accurate for 3D cracks. Thus, it is necessary to use others factors to describe the out-of-plane constraint. The source of a change of the out-of-plane constraint is associated with the body size in the z-direction, i.e. the specimen thickness. Nakamura and Parks [15] described a numerical method using an interaction integral to determine the elastic in-plane T-stress along a three-dimensional crack front. The T_z factor introduced by Guo [16] is an important parameter to characterize the constraint effect accurately, which is essential to establish a three parameter dominated stress field, and offers a possibility to characterize the stress-state in a 3D cracked body. The authors of Refs. [17–26] presented a description of the thickness effect on the crack-tip constraint and, in turn, on the fracture toughness, on the basis of numerical and experimental results. They analyzed the application of T-stress components, the local triaxiality parameter h, the T_z, T_{zz}, *and* T_{33} factors and the second-order term amplitude A_2 to the 3D crack-front stress field. In particular, Shlyannikov et al. [25] and Matvienko et al. [26] presented a description of the thickness effect on the interaction between the in-plane and out-of-plane constraints for a 3D cracked body based on finite element analyses of both power-law hardening elastic–plastic and creep materials.

The engineering application of the fracture mechanics of solids to real cracked structures requires an appropriate parameter to quantify the crack tip constraint. Moreover, practical structural components have finite thicknesses, and the stress–strain state changes between plane stress and plane strain. From a practical point of view, the most useful approach for assessing the fracture resistance of materials, components and structures would involve one common parameter, which, unlike the two parameter models and the higher order term solutions, would preserve the one-term representation.