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Intrinsic dissipation of a modular anisotropic damage model: Application to concrete under impact

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ABSTRACT

Based on the mathematical proof of the positivity of the dissipation due to anisotropic damage, different numerical scheme to compute such a dissipation in concrete structures are proposed. The positivity of intrinsic dissipation for the considered modular anisotropic damage model is first checked in the case of Willam non-proportional loading test, then checked for different impact tests on concrete structures. Both the consequences of the modeling of the strain rate effect in tension (from visco-damage) and of the damage deactivation (micro-cracks closure) for alternated loadings are studied.

1. Introduction

Damage mechanics is a powerful tool to handle micro-cracking and failure of quasi-brittle materials and structures. The corresponding constitutive models, even with induced anisotropy, are now most often implemented with localization limiters, i.e. either in a nonlocal form [1,2] or as visco-damage models [3–5] or both [6,7]. This allows to gain some numerical robustness as mesh independency of the converged finite element solution.

Numerical robustness is nevertheless still difficult to obtain when induced anisotropic damage is considered, i.e. when the state of damage is represented by a tensorial variable [10,11,20]. The key point of a thermodynamics consistent modelling is to ensure the positivity of the intrinsic dissipation [20], and in particular the positivity of the dissipation due to damage.

In case of isotropic (scalar) damage variable D, if Y stands for the (positive) thermodynamics force associated with D, the dissipation due to damage writes $\dot{\mathcal{D}} = Y\dot{D}$ so that one simply has to enforce $\dot{D} \geqslant 0$ from the damage evolution law [21] in order to satisfy the positivity of the dissipation.

In case of anisotropic damage, two cases arise:

(a) the damage evolution law is written in the framework of standard generalized materials [22], i.e. damage is proportional to the tensorial (positive) thermodynamics force $\mathbf{Y}$ or to $\mathbb{J} : \mathbf{Y}$ with $\mathbb{J}$ a positive definite fourth order tensor [23,24]. One has $\dot{\mathcal{D}} = \dot{\lambda}\mathbf{Y} : \mathbb{J} : \mathbf{Y} \geqslant 0$, if $\dot{\lambda}$ stands for the (positive) damage multiplier.
The drawback of such a "standard" modelling is the difficulty to conceptualize the anisotropy represented in the different loading cases and the number of material parameters introduced as components J_{ijkl}.

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(b) a practical possibility for quasi-brittle materials is to consider damage but also its anisotropy governed by the extensions [12,25] setting for example the damage rate proportional to the positive part of the strain tensor. Such a modelling is non standard, the positivity of the dissipation is not guaranteed for any models and has to be mathematically proven [26].

One considers here a modular anisotropic damage model, non standard (case b), for which the proof of the positivity of the dissipation can be mathematically given [27]. The proof, recalled in Section 2.4, avoids the calculation of the tensorial thermodynamics force **Y** associated with second order damage **D**. Using this feature, one proposes in present work differents ways to determine within finite elements computations the dissipation for the considered anisotropic damage model. As pointed out in [30], the quantification of the energy dissipated in the rupture of structures made of quasi-brittle materials gives complementary information to the usual damage maps.

2. Thermodynamics of anisotropic damage

A state of micro-cracking at the Representative Volume Element (RVE) scale can be represented by a thermodynamics state variable, namely the damage variable. For instance, in concrete-like materials the micro-cracks are mainly orthogonal to the loading direction in tension and parallel to the load in compression. A second order damage tensor is then often considered to be able to represent such an induced anisotropic damage pattern [10–19,29]. The choice to use of a second order tensor for damage – **D** of components D_{ij} is made here. Such a choice is not a necessity (a fourth order tensor can be used for instance [23,28]) but it has proven very helpful to solve the incompatibility with thermodynamics of the consideration of two damage variables, one for tension and one for compression: damage represents the state of micro-cracking whatever the sign of the loading so that only one damage variable represents the microcracking pattern [18].

Hydrostatic or mean damage is then

$$D_H = \frac{1}{3}\ \mathrm{tr}\ \mathbf{D} \tag{1}$$

In order to simplify further expressions, a specific integrity tensor **H** is defined,

$$\mathbf{H} = (1-\mathbf{D})^{-1/2} \tag{2}$$

where to take power α (here $-1/2$) of a symmetric tensor, one makes it first diagonal, one takes then the power α of the diagonal components and one finally turns back the tensor obtained in the initial working basis.

2.1. State potential and state laws

Following [27,31], the permanent strains are neglected and the free enthalpy density is written

$$\rho\psi^\star = \frac{1+\nu}{2E}\ \mathrm{tr}\left[\mathbf{H}\boldsymbol{\sigma}^D\mathbf{H}\boldsymbol{\sigma}^D\right] + \frac{1-2\nu}{6E}\left(\frac{\langle\ \mathrm{tr}\ \boldsymbol{\sigma}\rangle_+^2}{1-\eta D_H} + \langle\ \mathrm{tr}\ \boldsymbol{\sigma}\rangle_-^2\right) \tag{3}$$

with E the Young modulus, ν the Poisson ratio, ρ the density, η the hydrostatic sensitivity parameter, and where $(\cdot)^D = (\cdot) - \frac{1}{3}\ \mathrm{tr}(\cdot)\mathbf{1}$ stands for deviatoric part of a tensor, $\langle\cdot\rangle_+$ and $\langle\cdot\rangle_-$ respectively for positive and negative parts of a scalar, $\langle x\rangle_+ = \max(x,0)$ and $\langle x\rangle_- = \min(x,0)$.

The elasticity law derives from state potential (3),

$$\epsilon = \rho\frac{\partial\psi^\star}{\partial\boldsymbol{\sigma}} = \frac{1+\nu}{E}\left(\mathbf{H}\boldsymbol{\sigma}^D\mathbf{H}\right)^D + \frac{1-2\nu}{3E}\left(\frac{\langle\mathrm{tr}\ \boldsymbol{\sigma}\rangle_+}{1-\eta D_H} + \langle\mathrm{tr}\ \boldsymbol{\sigma}\rangle_-\right)1 \tag{4}$$

The thermodynamics force associated with damage **D** is

$$\mathbf{Y} = \rho\frac{\partial\psi^\star}{\partial\mathbf{D}} \tag{5}$$

Its calculation is not an easy task (see Appendix A). Key-point of present work, one will avoid to have to calculate such a derivative with respect to **D**: one will only use the simpler derivative with respect to tensor **H**, with the property

$$\frac{\partial}{\partial\mathbf{H}}\ \mathrm{tr}\left[\mathbf{H}\boldsymbol{\sigma}^D\mathbf{H}\boldsymbol{\sigma}^D\right] = 2\boldsymbol{\sigma}^D\mathbf{H}\boldsymbol{\sigma}^D \tag{6}$$

which defines a symmetric second order tensor.

The elasticity law coupled with anisotropic damage (4) can be inverted as

$$\boldsymbol{\sigma} = \frac{E}{1+\nu}\left(\mathbf{H}^{-1}\boldsymbol{\epsilon}\mathbf{H}^{-1} - \frac{(1-\mathbf{D}):\boldsymbol{\epsilon}}{3-\mathrm{tr}\ \mathbf{D}}(1-\mathbf{D})\right) + \frac{E}{3(1-2\nu)}\left[(1-\eta D_H)\langle\mathrm{tr}\ \boldsymbol{\epsilon}\rangle_+ + \langle\mathrm{tr}\ \boldsymbol{\epsilon}\rangle_-\right]\mathbf{1} \tag{7}$$

with then $\mathbf{H}^{-1} = (1-\mathbf{D})^{1/2}$.