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Generalized Kelvin model for micro-cracked viscoelastic materials

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ABSTRACT

The aim of this paper is to model the viscoelastic properties of micro-cracked materials based on the homogenization micro–macro approach. The isotropic case with random orientation distribution of micro-crack in Burgers nonageing linear viscoelastic solid was previously modeled. This study develops an alternative generalized Kelvin viscoelastic model (GKM) for fractured viscoelastic materials. The use of the same GKM of the non-cracked materials to model the viscoelastic properties of the micro-cracked materials is an approximation. This is a new technique to avoid the complexity of the inverse Laplace–Carson (LC) transform. This approximation is carried out in short and long term behavior in the LC space and is validated in transient situation with exact solution obtained from the inverse LC transform for simple loading condition. This paper focuses on the isotropic case with random orientation distribution of open cracks.

1. Introduction

In many materials like concrete or rocks, the presence of micro-cracks is permanent and affects the behavior of the materials [1,4,6,8,14,15]. For the case of concrete, the impact of micro-cracks on viscoelastic properties is attracting a good deal of attention nowadays. The micro–macro approach is largely developed to deal with the case of heterogeneous composite comprising micro-fractured materials [3,16]. The coupling between the micro–macro technique and the LC transform [5,12] allows modeling the evolution of the viscoelastic properties of linear nonageing viscoelastic heterogeneous materials [7,9,10].

The effective behavior of the heterogeneous media is firstly obtained in the LC space with help of the homogenization techniques and the inverse LC transform is usually employed to return to the real space [9]. The inverse LC is analytically complex and sometimes impossible. In such a case the numerical or semi-numerical methodologies are needed. The latter is also complex and consumes huge computational simulation time. This paper presents a very useful technique to avoid the complexity of the inverse LC transform by developing a set of explicit formulas for effective viscoelastic properties of heterogeneous media.

For the case of isotropic damage viscoelastic materials with random distribution of micro-cracks, based on LC transform and on the linear relationships between the jumps of displacements across the cracks and the macroscopic stress, Nguyen et al. [11] developed explicit formulas for the evolution of eight viscoelastic parameters of the Burgers model as functions of the crack density parameter.

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Nomenclature

$\mathbb{C}$	fourth order stiffness tensor
$\boldsymbol{\Sigma}$	second order macroscopic stress tensor
$\mathbf{E}$	second order macroscopic strain tensor
$\mathbf{1}$	second order unit tensor
p	Laplace–Carson variable
t	time
τ	characteristic time
k	bulk modulus
μ	shear modulus
ν	Poisson ratio
η	viscosity
ϵ	crack density parameter
$*$	exponent for the variables in Laplace–Carson space
K	index for the Kelvin part of the viscoelastic rheological model
s	exponent for the spherical part
d	exponent for the deviatoric part
o	index/exponent for short term behavior
∞	index/exponent for long term behavior
hom	exponent for homogenized values

However, in many cases the viscoelastic behavior of the materials is modeled by the GKM. The alternative development of [11] for the more complicated viscoelastic behaviors such as the GKM is therefore necessary.

As shown in [11], the LC transform yields the state equation of linear nonageing viscoelastic materials which relates the stress and the strain tensor into the linear form $\boldsymbol{\sigma}^* = \mathbb{C}^*(p) : \boldsymbol{\varepsilon}^*$ where the exponent $*$ stands for the LC transform and $\mathbb{C}^*(p)$ is the apparent stiffness tensor in the LC space. For the case of isotropic viscoelastic solid, $\mathbb{C}^*(p)$ depends on two scalars: apparent bulk modulus $k^*(p)$ and apparent shear modulus $\mu^*(p)$. The apparent Poisson coefficient $\nu^*(p)$ is commonly defined as:

$$\nu^* = \frac{3k^* - 2\mu^*}{6k^* + 2\mu^*} \tag{1}$$

Considering the so-called stress-based dilute scheme [2] of a single penny-shaped crack in an infinite elastic medium of stiffness $\mathbb{C}^*(p)$ with a remote stress state $\boldsymbol{\Sigma}^*$. Nguyen et al. [11] developed the homogenized bulk and shear moduli of the micro-cracked viscoelastic materials in LC space:

$$\begin{aligned} k^{hom^*} &= \frac{k^*}{(1+\epsilon Q^*)} \quad \text{with} \quad Q^* = \frac{16\left(1-\nu^{*2}\right)}{9(1-2\nu^*)} \\ \mu^{hom^*} &= \frac{\mu^*}{(1+\epsilon M^*)} \quad \text{with} \quad M^* = \frac{32(1-\nu^*)(5-\nu^*)}{45(2-\nu^*)} \end{aligned} \tag{2}$$

where k^*, μ^* and ν^* are respectively the apparent bulk, shear moduli and the apparent Poisson ratio (see (1)) in LC space of the solid phase (non-cracked); ϵ is the crack density parameter which is defined by: $\epsilon = Na^3$ with α is the radius of the cracks (the cracks are supposed to have the same radius) and N is the number of the cracks per unit volume of the micro-cracked material. Note that this solution is for the case of open cracks which is studied in this paper.

To avoid the complexity due to the inverse LC transform, the idea shown in [11] is to approach the effective viscoelastic behavior of the micro-cracked material by the same viscoelastic model of the solid phase and by approaching k^{hom^*} and μ^{hom^*} in the LC space by carrying out the short and long term behaviors. The idea is to develop k^{hom^*} and μ^{hom^*} in polynomial series of the LC variable p in the vicinity of $p = 0$ (long term) and of $p = \infty$ (short term) (see also [17,18]). This development depends on the viscoelastic behavior of the solid phase.

The development in [11] is limited to the case of micro-cracked Burgers materials. To develop this concept for more complex viscoelastic behavior such as the GKM, the main idea is to consider (1) and (2) with k^* and μ^* are functions of the viscoelastic properties of the solid phase that depend on the viscoelastic model chosen.

Firstly, Section 2 provides a simple three elements rheological model (standard model for solid, Fig. 1). This model is then generalized in Section 3 for $2n+1$ elements of n Kelvin's systems (a spring in continue with a dash-pot) in parallel with a spring (Fig. 2).

2. Three elements model

First we consider a three elements rheological model (two springs and one dash-pot) as shown in Fig. 1. This is a spring in continue with a Kelvin system (a second spring in parallel with a dash-pot). Considering the isotropic behavior, each spring is