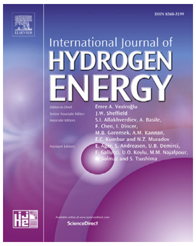
Available online at www.sciencedirect.com

ScienceDirect

journal homepage: www.elsevier.com/locate/he



Error of Darcy's law for serpentine flow fields: An analytical approach

Xuyang Zhang, Xu Zhang, Hidetaka Taira, Hongtan Liu *

Clean Energy Research Institute, College of Engineering, University of Miami, Coral Gables, FL 33146, USA

ARTICLE INFO

Article history:
Received 26 December 2017
Received in revised form
19 January 2018
Accepted 11 February 2018
Available online xxx

Keywords:
Darcy's law
Modified Darcy's law
Gas diffusion layer
Pressure drop
Under-land cross-flow
PEM fuel cell

ABSTRACT

Serpentine flow fields and other flow fields with partial under-land cross-flow are commonly used in various energy devices, such as proton exchange membrane (PEM) fuel cells and redox flow batteries, due to their higher mass transfer rate to reaction sites and better product removal capability. Accurately predicting the under-land cross-flow rate and pressure drop in such flow fields is crucial in flow field design optimizations. Darcy's law is the most commonly used model in predicting the under-land cross-flow and pressure drop in such flow fields. However, since the Darcy's law neglects inertial effect, its validity in different designs and operating conditions needs to be carefully studied. In this work, mathematical models for a serpentine flow field are developed based on both the Darcy's law and a modified Darcy's law that includes the inertial effect. Both models are solved and analytical solutions are obtained. The predicted pressure drops and under-land cross-flow rates from the two models are compared with experimental data and the results show that under some conditions, both the Darcy's law and the modified Darcy's law can predict pressure drop and under-land cross-flow rate reasonably well. However, under other conditions the Darcy's law can result in significantly large errors in predicting both pressure drop and under-land cross-flow rates. Further studies provide the variations of errors from the Darcy's law with different parameters, including channel length, gas diffusion layer (GDL) thickness, land width, inlet flow rate, GDL permeability and GDL inertial coefficient.

Introduction

Serpentine flow fields are widely used in various energy devices including fuel cells and redox flow batteries due to their unique advantages. For instance, in proton exchange membrane (PEM) fuel cells and redox flow batteries, pressure difference between two adjacent channels induces under-land convection (cross-flow) through the porous gas/liquid diffusion layer (GDL/LDL), enhancing mass transfer to the catalyst layer and improving removal capabilities of water or other products, resulting in higher and more uniform current distributions [1–5]. Ouellette et al. [6] found the under-land cross-flow rate plays an important role in maintaining an uniform reactant concentration distribution in direct methanol fuel cells. However, too high under-land cross-flow rate could lead to membrane dehydration and a higher pumping power resulting in decreases in the overall fuel cell system efficiency [7,8]. Also, too high under-land cross-flow can lead to a high methanol crossover flux in direct methanol fuel cells

---

* Corresponding author.
E-mail address: hliu@miami.edu (H. Liu).

https://doi.org/10.1016/j.ijhydene.2018.02.070

Please cite this article in press as: Zhang X, et al., Error of Darcy's law for serpentine flow fields: An analytical approach, International Journal of Hydrogen Energy (2018), https://doi.org/10.1016/j.ijhydene.2018.02.070

Nomenclature

A	Channel cross-section area (m^2)
β	GDL inertial coefficient (m^{-1})
C_1	Shape constant
C_f	Friction coefficient along the channel
D	Hydraulic diameter of the channel cross-section (m^2)
η	Under-land cross-flow rate error (%)
δ	GDL thickness (m)
δ_0	GDL original thickness (m)
ε_0	GDL original porosity
k	GDL permeability (m^2)
K_U	Effective coefficient of the U-turn (m^2)
L	Channel length (m)
μ	Dynamic viscosity ($kg\ m^{-1}\ s^{-1}$)
P	Pressure ($kg\ m^{-1}\ s^{-2}$)
ΔP_{ch}	Pressure drop along the channel ($kg\ m^{-1}\ s^{-2}$)
ΔP_U	Pressure drop at U-turn ($kg\ m^{-1}\ s^{-2}$)
P_{in}	Inlet pressure ($kg\ m^{-1}\ s^{-2}$)
P_{out}	Outlet pressure ($kg\ m^{-1}\ s^{-2}$)
Q_{cr}	Under-land cross-flow rate ($m^3\ s^{-1}$)
Q_{cr}^D	Under-land cross-flow rate from Darcy's law ($m^3\ s^{-1}$)
Q_{cr}^{MD}	Under-land cross-flow rate from modified Darcy's Law ($m^3\ s^{-1}$)
Q_{in}	Inlet flow rate ($m^3\ s^{-1}$)
Q_{out}	Outlet flow rate ($m^3\ s^{-1}$)
Q_U	U-turn flow rate ($m^3\ s^{-1}$)
Q_{cr}^D	U-turn flow rate from Darcy's law ($m^3\ s^{-1}$)
Q_{cr}^{MD}	U-turn flow rate from modified Darcy's Law ($m^3\ s^{-1}$)
Re	Reynolds number
u	Fluid velocity in channel ($m\ s^{-1}$)
v	Under-land cross-flow velocity ($m\ s^{-1}$)
v_0	Maximum under-land cross-flow velocity ($m\ s^{-1}$)
w	Land width (m)

[9]. Latha and Jayanti [10] shown that the pressure drop predicted without considering the under-land cross-flow was much higher than the experimental data for flow with high Reynolds numbers. Therefore, to ensure a good balance between pumping power, water removal capability and membrane hydration, accurate predictions of both the under-land cross-flow rate and the pressure drop are critical.

Generally, Darcy's law is applied to calculate the under-land cross-flow to evaluate the cell performance in serpentine flow fields [5,10–19]. Based on Darcy's law, a linear relationship between the under-land cross-flow rate and the pressure difference between two adjacent channels can be obtained [20,21]. Feser et al. [17] predicted that the under-land cross-flow rate was higher in a single serpentine flow field with longer flow channel in their modeling studies. Pharoah [18] found that the under-land cross-flow rate became quite significant in serpentine flow fields when the GDL permeability was over 10^{-13} m^2. Ye et al. [19] also found that the effect of under-land cross-flow became important when the GDL permeability was over 10^{-13} m^2. Moreover, Akhtar et al. [22] applied Darcy's law to investigate the effect of channel and land width on fluid transport at the cathode side, such as liquid water saturation distribution in the porous layers. Zamel et al. [23] used Darcy's law to study the effect of Teflon treatment in the GDL on capillary pressure and mass transfer loss. Rahimi-Esbo et al. [24] utilized Darcy's law to evaluate fuel cell performance with modified serpentine flow fields and found that the performance is enhanced significantly at high current densities for 2-1-channel serpentine flow field (two channel serpentine at the beginning converted to one channel at the end). Haghayegh et al. [25] employed Darcy's law to predict fuel cell performance with different MEAs in a single serpentine flow field.

Although Darcy's law is widely used in PEM fuel cells, its accuracy is low when under-land cross-flow rate is high, due to the neglect of the inertial effect [26]. The pressure difference between two adjacent channels was not a linear function of the under-land cross-flow rate through the GDL when the under-land cross-flow rate was relatively high [27,28]. Gostick et al. [29] found that Darcy's law was accurate only when the under-land cross-flow rate through the GDL was low enough. Zeng and Grigg [30] used a non-Darcy's term to represent the inertial effect. Ouellette et al. [6], Kim et al. [31], and Houser et al. [32] applied modified Darcy's law that includes the inertial effect to study the effect of cathode flow fields on the performance of direct methanol fuel cells, PEM fuel cells, and vanadium redox flow batteries, respectively, but only Kim et al. [31] specified the inertial coefficient in the GDL. Taira and Liu [28] directly measured the effective permeability and inertial coefficient of carbon cloth in PEM fuel cell and found that both the effective permeability and inertial coefficient increased when the land width decreased. Their results shown that under high under-land cross-flow rate, inertial effect was not negligible, and pressure drop was no longer a linear function of the under-land cross-flow rate through the GDL. Without considering the inertial effect, the under-land cross-flow rate through the GDL will be over-predicted, and the pressure drop will be under-predicted. The error in the under-land cross-flow rate predicted by the conventional Darcy's law will also cause errors in predicting both water removal ability and current density in under-land region.

In summary, accurate predictions of the under-land cross-flow rates and pressure drops are crucial in predicting the performance of both fuel cells and flow batteries. Therefore, the objective of this study is to analyze the errors of Darcy's law in predicting both the pressure drops and the under-land cross-flow rates. Two mathematical models, one based on the Darcy's law and the other based on the modified Darcy's law, are developed and solved analytically. The analytical solutions of pressure drop and under-land cross-flow rate from the two models, as well as experimental results, are used to study the errors of Darcy's law.

Model developments and solutions

Fig. 1 shows a single serpentine flow field with two adjacent channels. As a large portion of inlet gas flows along the channel, some portion of the gas flows from the upstream

Please cite this article in press as: Zhang X, et al., Error of Darcy's law for serpentine flow fields: An analytical approach, International Journal of Hydrogen Energy (2018), https://doi.org/10.1016/j.ijhydene.2018.02.070