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Improved stress intensity factors for selected configurations in cracked plates

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In this paper improved stress intensity geometry factors are determined for four key geometric configurations. They were developed using a p-version finite element method program. Two-dimensional uniaxially loaded plates are investigated, with either: an edge crack, a crack approaching a hole, or a crack propagating from a hole after ligament failure. The three-dimensional problem of a through crack in an integral stiffener approaching a junction, under uniaxial tension, is also considered. The resulting normalised stress intensity factor data are provided as compact equations or presented in tabular form.

1. Introduction

Damage Tolerance Analysis (DTA) of aircraft structures is a primary tool in managing aircraft safety. One of the fundamental inputs to a DTA is the stress intensity factor (K), which is used to determine the crack growth life as well as the critical crack length. The general equation used to define the stress intensity is:

$$K = \sigma\beta\sqrt{\pi a} \tag{1}$$

where σ is a reference stress, a is the crack length and β is the beta factor. The beta factor is considered as the normalised stress intensity factor and accounts for geometry effects. The beta factor for common simple geometries is available from handbooks [1–5], whilst many DTA software codes also include such beta factor solutions (e.g. [6,7]). Rearranging Eq. (1), beta factor is given by:

$$\beta = \frac{K}{\sigma\sqrt{\pi a}} \tag{2}$$

It is known that in such handbooks (and the related underpinning journal articles), the accuracy and range of beta factor values available can be variable. These solutions are based on a mixture of analytical and numerical approaches. One reason for the inaccuracies is that certain solutions were developed decades ago where the capability of numerical methods, such as Finite Element Analysis (FEA), were significantly limited. Some solutions have however been revised over the years. Since publication of the historic work there has been continual advancement in FEA capability, especially for three dimensional (3D) problems. For example, p-element methods [8] can be applied to achieve more accurate beta factor solutions. An extensive application of such p-element modelling is given by Fawaz and Andersson [9] for plates with corner cracks at a hole.

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Nomenclature

a	length of edge crack or half-length of imbedded crack
b	distance between hole edge and crack centre
c	distance between the centre of the hole and the centre of the crack
E	Young's modulus
e	half-width of notch at plate edge
h	half-height of plate
h_s	stiffener height
K	stress intensity factor
K_t	stress concentration factor
L	nominal notch length
p	stiffener pitch
R	radius of hole or radius of notch end
s	interaction zone
t_p	plate thickness
t_s	stiffener thickness
u_y	uniform displacement in y-direction
w	width of plate
$\tilde{y}$	normalised position on crack front
β	beta factor, normalised stress intensity factor
δ	displacement
ν	Poisson's ratio
σ	reference stress

Based on a review of the literature and the authors' experience in developing and applying beta solutions for airframe life assessment, four generic cases were identified as needing improvement. These cases are uniaxially loaded plates with either: (i) a through edge crack, (ii) a through crack approaching a hole, (iii) a through crack propagating from a hole after ligament failure, or (iv) a through crack in an integral stiffener approaching a junction.

It is important to note that to achieve results for more complicated practical geometries, compounding of multiple generic handbook solutions is typically used. For example, prior work by the authors demonstrates the use of compounding to analyse C-130J-30 airframe configurations such as cracking in: skin panels stiffened by hat stringers, 'L' section spar caps, and panels with integral stringers, [10]. This work also includes some preliminary and limited results for generic cases (i), (ii) and (iv).

The present paper focuses on the four cases listed above. The geometries of cases (i) to (iii) are 2D and have been investigated in the literature to various degrees [1–7,11–13]. Whereas case (iv) is three dimensional. Initially this paper extends the accuracy and range of values previously provided for cases (i) and (ii). Then new solutions are determined for cases (iii) and (iv), where to the authors' knowledge, no published solutions are available. For these two cases, current practice usually involves the use of approximate geometries.

To improve the usefulness of the new FE generated beta factors, a wide range of key parameters were considered. Compact equations are given for the cases of an edge crack, and a crack from a hole after ligament failure; the remaining two solutions obtained are presented in tabular form. The background for each of the four geometric cases is provided more fully, along with comparisons where possible with literature results.

2. Methods and assumptions

2.1. Finite element modelling

The StressCheck® commercial FE software package, Version 8.0.1 [14] was used for all 2D and 3D FEA. This code uses the variable order polynomial elements (p-element) approach, and can be used to determine the Mode I stress intensity for cracked components. Such p-version software can reduce the FEA discretisation error for a fixed mesh, by automatically increasing the p-order of the element shape function and displacement function. This is in comparison to h-version FE software, which requires mesh refinement to reduce the error. Typically, the maximum order polynomial in h-version FEA is two, while p-element FEA allows up to eighth order polynomials. The use of p-elements allows a relatively coarse geometric mesh to be used, enabling quick mesh creation with a high level of accuracy.

In the FEA undertaken here, the stress intensity factors are computed using the super-convergent contour integral method. Stress intensity factors are output as a standard result for linear elastic analysis where a crack boundary has been