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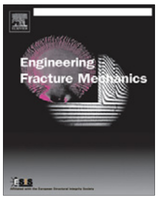

Incremental displacement collocation method for the evaluation of tension softening curve of mortar

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ABSTRACT

The tension softening curve (TSC), showing the relationship between the cohesive tensile stress and crack opening displacement, is the constitutive law of the cohesive crack model. Due to the difficulties in measuring local deformations around the crack tip, the TSC is usually determined inversely from the global responses such as load–deflection curve or load–crack mouth opening displacement curve of pre-notched specimens. However, the use of global responses alone in the inverse analysis usually causes problems that may affect the reliability and accuracy of the TSC which is basically a local material property. To overcome these limitations, an incremental displacement collocation method (IDCM) that is able to evaluate the TSC in a step-by-step manner is proposed in this paper. Both global and local responses of a pre-notched mortar beam, which are measured using an electronic speckle pattern interferometry technique, are used in the displacement collocation process. Furthermore, the finite element model (FEM) is utilized to simulate the response of the beam. The TSCs evaluated in this study are verified through the comparisons of the global and local displacements as well as the fracture energy. A tri-linear curve was found to be the best approximation of the TSC of mortar.

1. Introduction

Nonlinear fracture mechanics models have been proposed to explain the nonlinear fracture behavior of quasi-brittle materials, such as the cohesive crack model (CCM) or the fictitious crack model proposed by Hillerborg et al. [1]. According to the CCM, all the nonlinear behaviors in the fracture process zone (FPZ) are represented by a cohesive crack, and the crack propagation is dominated by the relationship of the cohesive stress versus crack opening, namely, the tension softening curve (TSC). The shape of the TSC has a significant influence on the computed results of a cracked beam in a finite element (FE) analysis; thus, a reliable estimation of the TSC is necessary.

Many efforts have been made to obtain the TSC. Ideally, the TSC can be obtained from uniaxial tension tests of the specimen, and some researchers [2] have produced uniaxial tension test configurations to analyze the tension-softening principles of concrete. However, the crack path may not be known a priori or the crack propagation may not be stable and symmetrical. Therefore, only the average value of the stress and crack opening can be obtained. It is difficult to make an accurate estimation of the TSC from a uniaxial tension test.

An alternative approach to evaluate the TSC is the inverse analysis based on the parametric fitting, which makes the numerical responses of a specimen agree with the experimental responses. Usually, notched beams or compact specimens are tested, and the experimental responses, such as load–deflection curve or load–crack mouth opening displacement

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Nomenclature

P	applied load
P_{max}	peak load
L	length of the beam
S	span of the beam
B	depth of the beam
t	thickness of the beam
a_0	initial notch depth
σ	cohesive stress
w	crack opening
x_1	x coordinate corresponding to the rear end of the cohesive crack
x_2	x coordinate corresponding to the front end of the cohesive crack
l_p	length of the cohesive crack
P_i	applied load at the ith step
σ_i	trial cohesive stress to be determined at the ith step
w_i	crack opening corresponding to the cohesive stress σ_i
l_{pi}	length of the cohesive crack at the ith step
σ_{max}	the maximum stress calculated
E	Young's modulus
E_c	elastic modulus obtained from cylinder test
f_t	tensile strength of the material
f_{cu}	cube compressive strength
f_{st}	tensile strength by splitting cylinder
w_c	characteristic crack opening
d_e	experimental displacement
δ_e	experimental mid-span deflection
$CMOD_e$	experimental crack mouth opening displacement
$CTOD_e$	experimental crack tip opening displacement
d_n	numerical displacement
δ_n	numerical mid-span deflection
$CMOD_n$	numerical crack mouth opening displacement
$CTOD_n$	numerical crack tip opening displacement
$CTOD_c$	critical crack tip opening displacement at the peak load
N	number of elements in the FEM
N_x	number of FE elements in x direction
N_y	number of FE elements in y direction
N_c	number of FE elements in the FPZ
G_F	fracture energy
A_0	area under the measured load–displacement curve

(CMOD) curve, are used in the inverse analysis. The numerical responses of the specimen are obtained from a numerical model. As an input of the numerical model, the TSC is prescribed and obtained from an optimization procedure. Different assumptions on the shape of the TSC, such as linear [1], bilinear, tri-linear [3] and exponential [4–6] shapes, have been proposed. The bilinear curve is believed to be a reasonable model and has been most widely used in practice. Another approach to determine the TSC using the inverse method without a prior assumption on the shape of the TSC is based on the poly-linear approximation introduced by Kitsutaka [7] and applied by Kurihara et al. [8]. The TSC is determined from a series of loading states by correlating the numerical and experimental responses. For both aforementioned approaches, only the global responses of the specimen, such as load–deflection or load–CMOD curve, are considered in the inverse analysis, taking no account of local responses.

The local responses of the beam were used in a hybrid inverse technique proposed by Shen and Paulino [9]. However, only the local response at one loading state in the post-peak stage was considered. The extracted TSC was only verified through the comparisons of the global response (load–CMOD curve) without checking the local responses at other loading states. Skocek and Stang [10] performed wedge splitting tests and inversely estimated the fracture parameters using the optically measured displacements. The softening curve is assumed to be piecewise and linear. The number of line segments in the softening curve has to be given a priori and the unknown parameters increase with the number of segments used.

An incremental displacement collocation method (IDCM) allowing an estimation of the TSC in a step-by-step manner is introduced in this study. At each numerical step, a trial cohesive stress forming the TSC is presumed and used along with the experimental crack opening to predict the cohesive stress in the FPZ. The trial cohesive stress is determined through the displacement collocation of the experimental displacements measured using the electronic speckle pattern interferometry (ESPI) technique and the numerical displacements from FE analysis.