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Stress intensity factor solutions for spot welds in square overlap parts of cross-tension specimens of different thicknesses and materials

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ABSTRACT

Closed-form stress intensity factor solutions for spot welds in square overlap parts of cross-tension specimens are investigated in this paper. The opposite outer edges of the square overlap parts of cross-tension specimens are under combined opening and counter bending conditions. The closed-form analytical structural stress solutions for a rigid inclusion in a square plate under the combined opening and counter bending conditions are first reviewed. Then, the J integral and stress intensity factor solutions for spot welds between square plates of different thicknesses and materials under the combined opening and counter bending conditions are developed in terms of the closed-form structural stress solutions for a strip model. The results of three-dimensional finite element analyses for spot welds between square plates of different thicknesses and materials under the combined opening and counter bending conditions are then presented. The analytical stress intensity factor solutions at the critical locations for spot welds based on the structural stress solutions with the equivalent and fitting coefficients are compared with the computational results. The results indicate that the computational stress intensity factor solutions agree well with those based on the structural stress solutions with only the equivalent coefficients. Based on the closed-form structural stress solutions, complete sets of the normalized in-plane stress intensity factor solutions at the critical locations of spot welds in square overlap parts of cross-tension specimens as functions of the ratio of the plate width to the spot weld diameter are presented for combinations of steel, aluminum and magnesium sheets and combinations of aluminum and copper sheets for convenient engineering applications.

1. Introduction

Resistance spot welding is widely used to join sheet metals in the automotive industry. The fatigue lives of spot welds have been investigated by many researchers. Due to the geometry of spot welds, crack tips or notch tips are presented along the nugget circumference. Stress intensity factor solutions for spot welds at the critical locations in various types of specimens have been developed to investigate the fatigue lives of spot welds. Pook [1,2] gave the maximum stress intensity factors for spot welds in lap-shear, coach-peel specimens, circular plate, and other bending dominant plate and beam configurations. Radaj [3] and Radaj and Zhang [4–6] established the foundation to use the structural stress solutions to determine the stress intensity factor solutions for spot welds. Zhang [7,8] obtained the stress intensity factor solutions at the crit-

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Nomenclature

t	square plate thickness
b	half specimen or square plate width
a	rigid inclusion or spot weld radius
l	effective specimen length
A, B, C, D	critical locations of a spot weld
r, θ	polar coordinates
σ, τ	in-plane normal structural stress and out-of-plane shear structural stress along the rigid inclusion circumference
$\widetilde{M}_c$	uniform constraint moment per unit length
F	resultant force
b'	equivalent radius
E, ν	Young's modulus and Poisson's ratio
$\widetilde{M}_{x'y'}$	uniform twisting moment per unit length
p_c, q_c	equivalent coefficients
c_τ	numerical coefficient
k_{cII}, k_{cIII}	fitting coefficients
b/a	ratio of the plate width to the rigid inclusion or spot weld diameter
σ_{rr}, $\sigma_{r\theta}$	radial and shear stress along the rigid inclusion circumference in the polar coordinate system
σ_x, τ_{xz}	normal and shear structural stresses in the Cartesian coordinate system
Γ	contour counterclockwise from the lower crack face to the upper crack face
ds	differential arc length of the contour Γ
$\mathbf{n}$	unit outward normal to the differential arc length ds
n_x	x component of the unit outward normal $\mathbf{n}$
$\mathbf{T}$	traction vector on the differential arc length ds
$T_i(=\sigma_{ij}n_j)$	components of the traction vector $\mathbf{T}(i, j = x, y, z)$
$\mathbf{u}$	displacement vector
u_i	components of the displacement vector $\mathbf{u}(i = x, y, z)$
W	strain energy density
G	shear modulus
J_{xy}	in-plane part of the J integral related to the in-plane normal stresses
$\delta = t_u/t_l$	thickness ratio
$\eta = E'_u/E'_l$	modulus ratio
E'	effective Young's modulus under plane strain conditions
J_3 or J_{III}	out-of-plane part of the J integral related to the out-of-plane shear stresses
J	J integral
$\xi = G_u/G_l$	shear modulus ratio
E^*, G^*	effective Young's and shear moduli for calculation of the J integral for interface cracks
ε	bimaterial constant
K_{eq}	in-plane equivalent stress intensity factor
K_1, K_2, K_3	mode I, II and III stress intensity factor solutions for spot welds joining dissimilar sheet metals
ω	angular quantity that can be found in Zhang [23] and Suo and Hutchinson [24]
$\kappa = 3 - 4\nu$	coefficient
α, β	Dundurs parameters
K_I, K_{II}, K_{III}	conventional mode I, II and III stress intensity factor solutions for spot welds joining similar sheet metals
Mg	for magnesium sheet
Al	for aluminum sheet
Fe	for steel sheet
Cu	for copper sheet
x, y, z	Cartesian coordinates
g_{kI}, g_{kII}	dimensionless geometric functions for similar spot welds
g_{k1}, g_{k2}	dimensionless geometric functions for dissimilar spot welds
g_{keq}	dimensionless geometric functions for spot welds

Superscripts and Superscripts

u, l	upper and lower plates
i, o	inner and outer surfaces of the plates
eff	for structural stress solutions with fitting coefficients
kc	for stress intensity factor solutions with fitting coefficients