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Bi-material V-notch stress intensity factors by the fractal-like finite element method

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ABSTRACT

The fractal-like finite element method (FFEM) is developed to provide stress intensity factor (SIF) values for bi-material notches. The displacement fields around a bi-material notch tip are derived and employed as global interpolation functions in the FFEM to transform the large number of nodal displacements in the singular region to a small set of generalised co-ordinates leading to direct computation of the SIFs and the coefficients of the higher order terms. Various numerical examples for bi-material crack and notch cases are presented. New results for bi-material notches are reported.

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1. Introduction

In recent years, there has been a lot of interest in computing the SIFs for a general notch (or corner), because the presence of notches may lead to crack initiation and sudden failure. Most of the research reported in the literature [1–5] deals only with isotropic homogenous notches based on the pioneering work presented by Williams [6]. For bi-material notches, there are hardly any results reported. Most research is about the behaviour of the eigenvalues and its computation rather than computing the SIFs [7–9]. That is simply due to the complexity of the bi-material case. However, SIFs were reported for a bi-material crack, which is a special case of a notch (the notch opening angle is zero) by some authors, such as Matsumto et al. [10], Yuuki and Cho [11] and Miyazaki et al. [12]. Williams [6] showed that the stress and displacement expressions around a notch tip can be written as eigenfunction series expansions. For a single material (isotropic homogenous) notch, the singular eigenvalues are always real. The singular eigenvalues are those which are less than one and they result in unbounded stresses. In the case of a two material (bi-material) notch, the singular eigenvalues could be real or complex numbers. This means that different eigenfunction series expansions have to be used for each case.

The FFEM was originally developed to compute the SIFs for crack problems [13,14]. Reddy and Rao [15] extended the method to analyse the shape sensitivity for a homogeneous isotropic crack. The current authors successfully developed the FFEM to compute the SIFs for an isotropic homogenous notch [16–20]. It should be noted that the two-level finite element method [13], the fractal two level finite element methods [14], the fractal finite element mixed-mode method [15], and the fractal-like finite element method [16–20] are identically the same. In fact, they are all fractal-like because the finite elements are truly fractal in the radial direction from the point of singularity, whereas in the circumferential direction in a layer, the elements are not fractal in nature. Therefore, the fractal self-similar nature of the mesh applies to the radial direction

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Nomenclature

a	crack/notch length
$\mathbf{a}$	set of generalised co-ordinates
A, B, a, b	generalised co-ordinates
$\mathbf{d}$	nodal displacement vector
$\mathbf{d}_r$, $\mathbf{d}_m$, and $\mathbf{d}_s$	nodal displacement vectors of nodes in regular region, master nodes, and in singular region
$\mathbf{d}_s^{1st,2nd,\ldots}$	nodal displacements of the nodes in the first layer, second layer, ... in the singular region
E	Young's modulus
$\mathbf{f}$	nodal force vector
$\mathbf{f}_r$, $\mathbf{f}_m$, and $\mathbf{f}_s$	nodal force vectors of nodes in regular region, master nodes, and in singular region
$\bar{\mathbf{f}}_s^{1st,inn}$	transformed nodal force vectors of the first layer and the inner layers in the singular region
G	shear modulus
H	plate height
$\mathbf{K}$	stiffness matrix
$\mathbf{K}_{rr}$, $\mathbf{K}_{mr}$, $\mathbf{K}_{mm}$, $\mathbf{K}_{ss}$, ...	partitioned stiffness matrices (r refers to regular region, m to master nodes, and s to slave nodes)
$\mathbf{K}_s^n$	partitioned stiffness matrix of the nth layer in the singular region
$\overline{\mathbf{K}}_s^{1st}$, $\overline{\mathbf{K}}_s^{inn}$	transformed partitioned stiffness matrices of the first layer and the inner layers in the singular region
K_I, K_{II}	stress intensity factors of mode I, II
K_c	complex stress intensity factor
i	complex unit $\left(\sqrt{-1}\right)$
j	integer variables
NL	number of layers in the singular region
NT	number of terms of eigenfunction series expansion
P_x, P_y	forces
R_s	radius of singular region
r, θ	polar co-ordinates
$\mathbf{T}_s^n$	transformation matrix of the nodal displacements of the nth layer in the singular region
$\mathbf{T}_s^{1st}$	transformation matrix of the nodal displacements of the first layer in the singular region
W	width of single-edge-notched plate
u_x, u_y	displacements in x and y directions
x, y	cartesian co-ordinates
z	complex variable
α	angle between notch face and x-axis
γ	notch opening angle
ϕ, ω	complex potentials
λ	eigenvalue
ν	Poisson's ratio
ρ	similarity ratio
σ	normal stress
τ	shear stress

towards the notch/crack tip but not necessarily to the hoop direction. For this reason, the authors prefer to refer to the topology as being fractal-like.

In this paper, the authors develop the FFEM to compute the SIFs for a bi-material notch. The stress and displacement fields are derived for real and complex eigenvalues. Then, the FFEM is developed by employing the displacement expressions as global interpolation functions to compute the SIFs for bi-material notch problems. To demonstrate the accuracy of the FFEM to compute bi-material notch SIFs, the SIF values for various bi-material notch examples are computed and compared to available published results and results computed using different numerical approaches.

2. Global interpolation functions for a bi-material notch

The stress and displacement functions of a bi-material notch as shown in Fig. 1 can be expressed using a complex variable approach as [21]:

$$\sigma_{xx}^j + \sigma_{yy}^j = 4\mathrm{Re}(\phi^{j\prime}(z)) \tag{1}$$

$$\sigma_{yy}^j - i\tau_{xy}^j = \phi^{j\prime}(z) + (z - \bar{z})\overline{\phi^{j\prime\prime}(z)} + \overline{\omega^{j\prime}(z)} \tag{2}$$

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