Engineering Fracture Mechanics 102 (2013) 334–347

Contents lists available at SciVerse ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Stress intensity factors and T-stress for an edge interface crack by symplectic expansion

Zhenhuan Zhou [a,*], Xinsheng Xu [a], Andrew Y.T. Leung [b], Yi Huang [a]

[a] State Key Laboratory of Structure Analysis of Industrial Equipment and Department of Engineering Mechanics, Dalian University of Technology, Dalian 116024, PR China

[b] Department of Civil and Architectural, City University of Hong Kong, Hong Kong

ARTICLE INFO

Article history:
Received 1 November 2012
Received in revised form 23 February 2013
Accepted 12 March 2013

Keywords:
Symplectic method
Interface crack
Complex stress intensity factor
T-stress

ABSTRACT

An analytical method is presented for finding the complex stress intensity factors (SIFs) and T-stress at an edge bi-material interface crack. A Hamiltonian system is first established by introducing dual (conjugate) variables of displacements and stresses whose solutions are expanded in terms of the symplectic series. With the aid of the adjoint symplectic orthogonality, coefficients of the series are determined by the boundary conditions along the crack faces and along the external geometry. Analytical solutions of SIFs and T-stress are obtained simultaneously. Numerical examples including the complex mixed boundary conditions are given. Factors influencing the SIFs are discussed.

1. Introduction

In recent years, bi-material composites and structures are being widely used. With the rapidly increasing usage of such systems, much attention has been paid to the interface at where failure often initiates. The fracture behavior at the interface between these dissimilar materials is a critical phenomenon that often discourages the safe and confident use of these composites and structures. Therefore, analysis of interface cracks is fundamental to our understanding of the initiation and propagation of free-edge cracks.

Analytical determination of the SIFs at interface crack is useful for better understanding of the subsequent crack propagation and damage. Williams [1] was the first to provide a theoretical framework for analyzing bi-material interface crack problems by an asymptotic analysis of the elastic field at the tip of an open crack. His solution was subsequently enriched by many other researchers [2–4]. After that, various methods for evaluating the SIFs of bi-material interface cracks were developed. Shi [5] applied the generalized variational approach to determine SIFs for interface crack in finite size specimens using eigenfunction expansion. Liu et al. [6] derived the SIF solutions for cracks in a bi-material anisotropic infinite strip using dislocation. Node and Xu [7] described the interface crack problem by singular integral equations using body force method. Numerical solutions of singular integral equations were also discussed. Pant et al. [8] modeled the material discontinuity at the interface by a jump function and then extended the element free Galerkin method (EFGM) to the analysis of interface cracks lying between two dissimilar materials.

With the rapid development of computer technology, numerical methods become important to find engineering solutions. These include the boundary element method (BEM) and the finite element method (FEM). However, due to the numerical flutruation at singularity, these numerical methods need to be improved. Ryoji and Sang-Bong [9] applied the BEM using

* Corresponding author. Tel.: +86 13842853461; fax: +86 0411 84708393.
E-mail address: zhouzh@dlut.edu.cn (Z. Zhou).

0013-7944/$ - see front matter
http://dx.doi.org/10.1016/j.engfracmech.2013.03.007

Nomenclature

(r, θ)	polar coordinates
E	Young's modulus
υ	Poisson's ratio
u_r, u_θ	displacements along the r- and θ-axes
σ_{ij}	components of stresses
ε_{ij}	components of strains
U	potential energy density
L	Lagrange function
H	Hamiltonian function
$\mathbf{H}$	Hamiltonian operator matrix
$\mathbf{q}, \mathbf{p}$	mutually dual vectors
$\mathbf{\Psi}$	full state vector
μ	eigenvalues
$\boldsymbol{\psi}^{(\alpha)}, \boldsymbol{\psi}^{(\beta)}$	α-, β-set eigenfunctions
$K_{\mathrm{I}}, K_{\mathrm{II}}$	Mode I, II stress intensity factors
σ_T	T-stress

Hetenyi's fundamental solution for interface crack problems. The SIFs for the interface crack were determined by extrapolation. Miyazaki et al. [10] presented a new BEM using M1-integral to analyze two-dimensional bi-material interface crack problems. Pan and Amadei [11] proposed a BEM formulation for 2D anisotropic elastic bi-materials in which the displacements are collocated on the outer boundary and the traction integral equations are collocated on one side of the crack surface. Matsumto et al. [12] developed a method for interface cracks based on energy release rates and the BEM sensitivity analysis. Hadjesfandiari and Dargush [13] determined the complex SIFs associated with interface cracks between dissimilar materials by a boundary element formulation using displacements and weighted tractions as primary variables. Serier et al. [14], Belytschko and Gracie [15] analyzed the interface cracks by the extended finite element method (XFEM). Bjerkén and Persson [16] calculated the complex SIFs by the combine use of the crack closure integral method and FEM. Khandelwal and Kishen [17] employed complex variable functions in the FEM to evaluate the SIFs and J integral. To analyze an interface crack between anisotropic materials, Ikeda et al. [18] proposed a numerical method based on energy release rate which is obtained by the virtual crack extension method using FEM. Nagai et al. [19] employed the M-integral and the moving least-square method to calculate the SIFs of a three-dimensional interface crack using FEM. Profant et al. [20] applied the technique of continuously distributed dislocation to calculate the generalized stress intensity. At the edge interface crack in a bi-material bonded dissimilar strip, Lan et al. [21] applied the FEM focusing on the elements at the crack tip to solve the SIFs. Later, Noda and Lan [22] produced proportional singular stress fields in FEM by superimposing specific tensile and shear stresses at infinity in order to calculate SIFs for bonded plate in an infinite domain.

All the above works are carried out under the Lagrange (potential energy) formulation and the analytical solutions are limited by material geometries and loads. To break the limits, Zhong and his associates [23,24] developed a new symplectic elasticity method for some basic problems in solid mechanics and elasticity. Both static and dynamic problems can be handled [25]. It solves systematically some previously unsolvable problems of rectangular thin plates in free vibration [26]. After that, Leung et al. [27] extended the method into the evaluation of SIFs. Yao and Hu [28] developed a novel singular finite element to study cracked plates having arbitrary traction acting on crack surfaces. Zhang et al. [29] used the method to analyze the singularities of bi-material bonded bodies.

In this paper, the symplectic expansion method is introduced for determining the SIFs and T-stress of an edge interface crack between dissimilar materials. Using the Hamilton principle of mixed energy, a set of Hamiltonian dual equations is obtained. The system is then solved analytically by the method of separation of variables. The resulting eigensolutions are symplectic adjoint orthogonormal to each other. Thus, the behaviors of the crack tip are completely described by the combinations of the eigenfunctions. The coefficients of the series are determined by the boundary conditions of at the crack surfaces and the outer geometric boundaries. Mode I and II SIFs are directly obtained by the first few terms of the series. Comparisons to classical solutions are given to validate the efficiency and accuracy. New results are also presented.

2. Basic formulations

The model of an interface crack between two dissimilar isotropic materials (M_1 and M_2) is shown in Fig. 1. E_1, υ_1 and E_2, υ_2 are Young's modulus and Poisson's ratio for materials M_1 and M_2 respectively. The interface crack is located on the common edge ($\theta = 0°$) in polar coordinates (r, θ) where the r-axis is along the radial direction with the origin at the crack tip.

The stress–strain relation in polar coordinates under the plane stress assumption is