Engineering Fracture Mechanics 77 (2010) 3462–3480

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

# Solutions of the second elastic–plastic fracture mechanics parameter in test specimens

Ping Ding, Xin Wang *

*Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa, Ontario, Canada K1S 5B6*

ARTICLE INFO

*Article history:*
Received 29 January 2010
Received in revised form 2 September 2010
Accepted 13 September 2010
Available online 19 September 2010

*Keywords:*
Elastic–plastic fracture mechanics
Constraint effect
*J*-integral
*A*-term
$A_2$-term
*Q*-factor
Crack in test specimen
Crack-tip fields
Three-term solutions

ABSTRACT

Extensive finite element analyses have been conducted to obtain solutions of the *A*-term, which is the second parameter in three-term elastic–plastic asymptotic expansion, for test specimens. Three mode I crack plane-strain test specimens, i.e. single edge cracked plate (SECP), center cracked plate (CCP) and double edge cracked plate (DECP) were studied. The crack geometries analyzed included shallow to deep cracks. Solutions of *A*-term were obtained for material following the Ramberg–Osgood power law with hardening exponent of $n = 3, 4, 5, 7$ and 10. Remote tension loading was applied which covers from small-scale to large-scale yielding. Based on the finite element results, empirical equations to predict the *A*-terms under small-scale yielding (SSY) to large-scale yielding conditions were developed. In addition, by using the relationships between *A* and other commonly used second fracture parameters such as *Q* factor and $A_2$-term, the present solutions can be used to calculate parameters $A_2$ and *Q* as well. The results presented in the paper are suitable to calculate the second elastic–plastic fracture parameters for test specimens for a wide range of crack geometries, material strain hardening behaviors and loading conditions.



## 1. Introduction

Characterizations of crack-tip stress/strain fields are the foundation of fracture mechanics. In classical elastic–plastic fracture mechanics (EPFM), *J*-integral is commonly used to set the amplitude of the crack-tip stress fields (i.e. the Hutchison–Rice–Rosengren (HRR) field) [1–3]. It has been well established that elastic–plastic fracture mechanics using *J*-integral works well only for crack-tip stress/strain fields that are under high constraint conditions. Under high constraint conditions, the *J*-dominance is maintained under large-scale yielding conditions and the HRR fields [2,3] characterize the crack-tip stress/strain fields. However, for cracks under low constraint conditions, as the external load increases from small-scale to large-scale yielding, *J*-dominance will be gradually lost, i.e. the local crack-tip stress/strain fields deviate from the HRR fields. Additional parameter is required, together with *J*-integral, to quantify the crack-tip stress fields.

Therefore, several two-parameter approaches for elastic–plastic crack-tip fields have been proposed to overcome the limitation of one-parameter *J*-based fracture mechanics approach. Li and Wang [4] as well as Sharma and Aravas [5] first proposed to use the amplitude of the second term in the asymptotic expansion for mode I plane-strain condition of power-law hardening material as the additional parameter. Betegon and Hancock [6], Al-Ani and Hancock [7] and Du and Hancock [8] confirmed that *T*-stress, which is the second term of Williams' expansion of the elastic crack-tip field [9], can be used as constraint parameter, together with *J*-integral, in describing elastic–plastic crack-tip fields for a variety of plane-strain cases, which is called the *J*–*T* approach. O'Dowd and Shih [10,11] suggested the *J*–*Q* approach based on the main feature of the

* Corresponding author. Tel.: +1 613 520 2600x8308; fax: +1 613 520 5715.
*E-mail address:* xwang@mae.carleton.ca (X. Wang).

0013-7944/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engfracmech.2010.09.007

**Nomenclature**

| | |
|---|---|
| $a$ | depth of crack in the test specimen |
| $a, b, c, d$ | coefficients of cubic equation for calculating constraint parameter $A$ from FEA results |
| $a_i, b_i, c_i, d_i$ | coefficients of expression on deviation of the asymptotic stress fields from the FEA stress solution for $i$th fitting point |
| $A$ | constraint parameter (second fracture parameter) in $J$–$A$ crack-tip fields |
| $A_0, A_1, A_2$ | amplitudes of three-term asymptotic expansion for $J$–$A$ or $J$–$A_2$ crack-tip fields |
| $A_2$ | constraint parameter (second fracture parameter) in $J$–$A_2$ crack-tip fields |
| $A_{SSY}$ | small-scale yielding FEA solution of constraint parameter $A$ |
| $d_1, d_2, d_3$ | coefficients of empirical formulas for predicting constraint parameter $A$ |
| $e_{ij}$ | coefficients of empirical expressions for calculating coefficients $d_1$, $d_2$, $d_3$ |
| $E$ | Young's modulus |
| $f_{adj}$ | adjusting factor in empirical formulas for predicting constraint parameter $A$ |
| $f_{ij}$ | non-dimensional angular function in Williams' series expansion for stress |
| $g_i$ | angular functions for displacement boundary conditions in small-scale yielding model |
| $h$ | coefficient of empirical expression for calculating adjusting factor $f_{adj}$ |
| $H$ | half length of the test specimens |
| $I_n$ | scaling integral depending on hardening exponent $n$ |
| $J$ | $J$-integral |
| $K_I$ | stress intensity factor for mode I |
| $L$ | characteristic dimension in $J$–$A_2$ crack-tip fields |
| $n$ | material hardening exponent |
| $n_r$ | number of elements in radial direction at crack tip core region |
| $n_\theta$ | number of elements in angular direction at crack tip core region |
| $Q$ | constraint parameter (second fracture parameter) in $J$–$Q$ crack-tip fields |
| $r$ | radius in polar coordinates at crack tip |
| $\bar{r}$ | dimensionless radius in polar coordinates at crack tip |
| $\bar{r}_i$ | dimensionless radius in polar coordinates at crack tip for $i$th fitting point |
| $r_p$ | radius of crack tip plastic zone |
| $R$ | maximum radius of the small-scale yielding model |
| $s, t$ | powers in $J$–$A$ crack-tip fields |
| $s_1, s_2, s_3$ | powers in $J$–$A_2$ crack-tip fields |
| $T$ | $T$-stress |
| $u_x, u_y$ | boundary displacement components in $x$ and $y$ directions of cartesian coordinates |
| $w_i$ | weight for the $i$th fitting point |
| $W$ | width of test specimen |
| $x, y$ | Cartesian coordinates |
| $\alpha$ | material coefficient in Ramberg–Osgood relationship |
| $\delta_i$ | deviation of the asymptotic stress fields from the FEA stress solution for $i$th fitting point |
| $\varepsilon_{ij}$ | strain components |
| $\varepsilon_0$ | yield strain |
| $\theta$ | angle in polar coordinates at crack tip |
| $\theta_i$ | angle in polar coordinates at crack tip for $i$th fitting point |
| $\kappa$ | elastic constant |
| $\mu$ | shear modulus |
| $\sigma$ | remote tension loading (stress) applied on the boundary of test specimen |
| $\sigma_{ij}$ | stress components |
| $\sigma_0$ | yield stress |
| $\sigma_{FEM}$ | stress value from finite element analysis |
| $\bar{\sigma}_{ij}^{(0)}, \bar{\sigma}_{ij}^{(1)}, \bar{\sigma}_{ij}^{(2)}$ | dimensionless angular stress functions in $J$–$A$ crack-tip fields |
| $\tilde{\sigma}_{ij}^{(1)}, \tilde{\sigma}_{ij}^{(2)}, \tilde{\sigma}_{ij}^{(3)}$ | dimensionless angular stress functions in $J$–$A_2$ crack-tip fields |
| $\sigma_L$ | limit load of test specimen |
| $\upsilon$ | Poisson's ratio |

elastic–plastic crack-tip stress fields. The second fracture parameter $Q$ is defined as the difference between the stresses in crack-tip region determined by numerical analysis and the HRR or small-scale yielding (SSY) stress fields.

Following the early work of Li and Wang [4] and Sharma and Aravas [5], Yang et al. [12,13] conducted a more complete and sophisticated analysis for higher order terms of asymptotic expansion of stress and displacement fields in crack tip. They derived a three-term expansion of crack-tip field with two fracture parameters: $J$-integral and a second fracture parameter