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Technical Note

Numerical solution for the T-stress in branch crack problem with infinitesimal branch length

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ABSTRACT

This paper investigates the T-stress in a branch crack problem with infinitesimal branch length. The branch crack is composed of a main crack and many branches. The ratio of the lengths for branch to main crack is very small. A singular integral equation method is suggested to solve the problem numerically and the stress intensity factor and T-stress can be evaluated immediately. Many computed results for T-stress under different conditions for branches are presented. It is found from the computed results that the interaction for T-stress among branches is very complicated.

1. Introduction

In an earlier year, Williams studied the stress distribution in the vicinity of a crack tip [1]. From the traction free condition along the crack face, an expansion form for stress distribution was obtained. In the expansion form, the first term has a relation with the stress intensity factor. The second term, or the non-singular term in the Williams's expansion, was regarded as the T-stress [2].

Many papers were devoted to evaluate the T-stress at the crack tip. A few of them are introduced below. It was proved that a limit of the difference of two normal stress components ahead of crack tip would give the T-stress [3]. Karihaloo and Xiao 2001 developed a hybrid crack element to evaluate the higher order terms including the T-stress in the vicinity of crack tip of a three-point bend beam [4]. Fett developed a Green's function to evaluate the T-stress for a line crack in a circular plate [5]. Using the dislocation distribution method, several T-stress problems were solved [6]. More recently, crack back position and crack front position methods were suggested to solve the T-stress problem [7].

Many researchers studied the problem for finding the stress intensity factors for the branch crack and kinked crack [8–15]. However, those methods may not be useful to study the T-stress solution in the kinked crack and the branch crack problem, particularly in the case of infinitesimal branch length.

For the case of infinitesimal kink length in a kinked crack, Li and Xu proposed approximate analytical formulas for calculating the T-stress as well as stress intensify factors [16]. Since those formulas are derived from an asymptotic solution, and the approximation of solution is not easy to estimate. Recently, a numerical procedure is suggested to examine the mentioned approximate analytical formulas [17]. An extended weight function procedure for T-stresses for slightly curved crack was suggested [18], and some computed results for T-stress for a fork configuration were obtained.

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This paper investigates the T-stress in branch crack problem with infinitesimal branch length. The branch crack is composed of a main crack and many branches. The length of main crack is denoted by "2*a*" and length of branches is denoted by "2*d*". The ratio of d/a is chosen in a rather small range, from 0.002, 0.004, 0.006, 0.008 to 0.01. A singular integral equation method is suggested to solve the problem numerically [12]. From the solution of the singular integral equation, the stress intensity factors (SIFs) and T-stress can be obtained immediately. In the numerical examples, the branch crack problems with two branches or three branches are studied. Many computed results for T-stress under different conditions for remote loading and branches were presented. Comparison between previous result and ours is also carried out. It is found from the computed results that the interaction for T-stress among branches is very complicated.

2. Nomenclatures used

To facilitate reading the article, the following nomenclatures are introduced.

T_A, T_B, T_C	T-stress at the branch tip *A*, *B* and *C*, receptively
G_A, G_B, G_C	non-dimensional T-stress at the branch tip *A*, *B* and *C*, respectively
$\varphi'(z)$, $\psi'(z)$	complex potentials
σ_N, σ_{NT}, σ_T	stress components
T_j	T-stress at the branch tip t_{Bj}
$T_{j(u)}$ and $T_{j(p)}$	T-stress at the branch tip t_{Bj}, from the uniform field and perturbed field, respectively
t	complex coordinate for integration
t_{oj}	complex value representing the observation point
t_{Bj}	complex value representing the *j*th branch tip
$D = D_1 + iD_2$	a concentrated dislocation
$z = x + iy$	complex variable
i	a unit imaginary value

3. Evaluation of SIFs and T-stress in the branch crack problem with infinitesimal branch length

For evaluating the stress intensity factors and T-stress at the branch crack tip, it is suitable to use the superposition method. The original problem is shown in Fig. 1a. Without losing generality, it is assumed that the remote loading is $\sigma_y^\infty = p$. The original field can be considered as a superposition of a uniform field and a perturbation stress field, which are shown by Fig. 1b and c, respectively.

For solving the problem for the perturbation field shown by Fig. 1c, we can place the dislocation distribution $g'_k(t)$ $(k = 1, 2, \ldots, N)$ along the *k*th branch, and a concentrated dislocation D ($D = D_1 + iD_2$) at the origin. Therefore, a singular integral equation is formulated. From the solution of the integral equation, The SIFs and T-stress at the branch tip can be evaluated immediately. The detailed formulation for this problem can be found from Appendix A.

4. Numerical examples

In order to study of the influence caused by small branch length, some numerical examples are given below. In the examples, we choose d/a = 0.002, 0.004, 0.006, 0.008 to 0.01. Particular attention is paid to the interaction for T-stress between main crack and branches. Those computed results are new, which may not be found in other references.

Example 1. In the first example, a branch crack problem with a main crack and two small branches is investigated (Fig. 2a). The horizontal branch has a length "2*a*" and two inclined branches have a length "2*d*" with the inclined angle θ. The remote tension is denoted by $\sigma_x^\infty = p$, or $\sigma_y^\infty = p$. In computation, $M_1 = 135$ (the number of divisions for integration used in the quadrature rule) is assumed for the main crack in the quadrature rule for the numerical solution of the integral equation, $M_2 = 5$ is assumed for two branches.

In the case of (1) d/a = 0.002, 0.004, 0.006, 0.008 and 0.01, (2) $\theta = 15°, 30°, 45°, 60°$ and $75°$ and (3) $\sigma_x^\infty = p$ (Fig. 2a), the computed results for the T-stress at the branch tips "*A*" and "*C*" are expressed as

$$T_A = G_A(d/a, \theta)p = T_B, \quad T_C = G_C(d/a, \theta)p \tag{1}$$

From computed results we see $G_C(d/a, \theta) \approx 1$, and those values do not need to present. The computed results for non-dimensional T-stress, or $G_A(d/a, \theta)$, are listed in Table 1 and Fig. 3. From the presented results we see that for all assumed d/a values, from 0.002, ... to 0.01, the curves are merged into to one curve. Alternatively speaking, under the remote loading $\sigma_x^\infty = p$, the $G_A(d/a, \theta)$ values are not sensitive to the ratio d/a.

Previously, Fett et al. [18] completed a computation for T-stress under following conditions: (1) a main edge crack with length "*a*" is in a rectangular plate, (2) the applied loading is $\sigma_x = p$ and (3) the ratio is $d/a = 1/450$. A comparison between Fett's results and ours is presented in Fig. 4. From the presented results, coincidence between two sources has been found.