Engineering Fracture Mechanics 76 (2009) 2202–2216

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Limit pressures of 90° elbows with circumferential surface cracks

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ARTICLE INFO

Article history:
Received 29 July 2008
Received in revised form 12 June 2009
Accepted 1 July 2009
Available online 8 July 2009

Keywords:
Circumferential cracks
Elbow
Finite element limit analysis
Limit pressure

ABSTRACT

This paper provides approximate limit pressure solutions for circumferential cracked elbows, resulting from small strain finite element limit analyses using elastic–perfectly plastic materials. Circumferential through-wall and constant-depth surface cracks of which the circumferential lengths are limited to 50% of the circumference are considered. Two locations along the longitudinal direction are considered; one in the centre of the elbow, and the other in the junction between the elbow and the attached straight pipe. Along the circumference, either extrados or intrados cracks are considered. It is found that limit pressures of circumferential cracked elbows are not affected by the presence of the circumferential surface crack, unless it is sufficiently deep and long. Moreover, normalized limit pressures with respect to un-cracked limit pressures decrease almost linearly with increasing the relative crack depth and length. Based on finite element results, approximate closed-form solutions for limit pressures are proposed.

1. Introduction

Plastic limit analysis of pressurized pipes with circumferential cracks has been an important issue in the field of structural integrity assessment, due to its importance in design and assessment. For instance, plastic loads obtained from plastic limit analyses can be directly used to estimate maximum load-carrying capacities, see e.g. Ref. [1]. Furthermore, based on the reference stress approach [2], it can be used to estimate non-linear fracture mechanics parameters such as J and C^* integrals (see for instance Refs. [3–5]). As pressure loading is fundamental for pressurized piping, knowledge on plastic limit pressures of cracked piping components is essential. For cracked straight pipes, plastic limit pressure solutions are widely available (see for instance Refs. [6–10]).

Typical pipeworks include not only straight pipes but also elbows, and thus plastic limit analyses of circumferential cracked elbows need to be performed. Despite the importance of internal pressure on design and assessment of elbows, most of exiting works have been for plastic limit analyses of cracked elbows under bending (see for instance Refs. [11–15]). Some works (for instance Ref. [16]) investigated plastic limit loads for elbows under combined pressure and bending, but internal pressure was treated as a base load and thus the pure internal pressure case was not considered in details. For un-cracked elbows, Goodall [17] presented an analytical limit pressure solution for an elbow. For circumferential cracked elbows under internal pressure, Yahiaoui et al. [18] performed finite element (FE) limit analysis but their cases were not sufficient to draw closed-form limit pressure solutions for cracked elbows. Another notable point is that all exiting works assume a circumferential crack in the centre of an elbow. It is natural, as the elastic stress analysis shows that the stress magnitude is the maximum in the centre of an elbow. On the other hand, elbows are typically butt-welded to straight pipes. As weldment is vulnerable to cracking, circumferential cracks can also occur in the junction between an elbow and the attached straight pipe. In this respect, plastic limit analysis also needs to be performed for circumferential cracks in the junction.

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doi:10.1016/j.engfracmech.2009.07.005

Nomenclature

a	crack depth
L	length of an attached straight pipe
P	internal pressure
P_L	limit pressure of a cracked elbow
P_o	limit pressure of an un-cracked elbow
P_o^s	limit pressure of a straight pipe, see Eq. (3)
R	bend radius
r	mean pipe radius
t	thickness of a pipe
λ	bend characteristic, = Rt/r^2
σ_o	limiting stress of an elastic–perfectly plastic material
φ	angular coordinate (see Fig. 1)
θ	half circumferential angle of a circumferential crack

In this paper, plastic limit pressures of circumferential cracked elbows are presented. The results are based on three-dimensional FE limit analyses using elastic–perfectly plastic materials. Circumferential through-wall and constant-depth surface cracks either at extrados or at intrados are considered. Furthermore two locations along the longitudinal direction of elbows are considered, either in the centre of the elbow or in the junction between the elbow and the attached straight pipe.

2. Finite element limit analyses

2.1. Geometry

Fig. 1a depicts a 90° elbow, considered in the present work. The mean radius and thickness of the pipe are denoted by r and t, respectively, and the bend radius by R, leading to non-dimensional variables, R/r and r/t. The non-dimensional bend characteristic should be also noted:

$$\lambda = \frac{Rt}{r^2} = \frac{(R/r)}{(r/t)} \tag{1}$$

To quantify the effect of the bend geometry on plastic loads, above non-dimensional variables were systematically varied. Mainly three different values of r/t, $r/t = 5$, 10 and 20, were considered, together with two values of R/r, $R/r = 2$ and 3. For selected cases, however, higher values of r/t and R/r were also considered to fully quantify the effects of r/t and R/r on plastic limit pressures. The piping system considered comprised the 90° bend and the attached straight pipe of length L (Fig. 1a). Introduction of the attached straight pipe is to minimize the end effect due to the applied loading. The effect of the length of the attached straight pipe, L, on plastic behaviour is found to be minimal, as long as it is longer than four time the pipe radius, $L = 4r$ [19]. In this paper, it was chosen to be twenty times the pipe radius, $L = 20r$.

Both circumferential through-wall and surface cracks were considered. The circumferential through-wall crack is characterized by its relative crack length, θ/π, where θ denotes the half crack length (Fig. 1b). The value of θ/π was systematically varied from $\theta/\pi = 0$ to $\theta/\pi = 0.5$, which would cover interesting ranges of crack in practical situations. Furthermore, when the value of θ/π is larger than $\theta/\pi = 0.5$, the crack closure phenomenon could occur, which is quite complex to analyse. For circumferential surface cracks, one additional geometric variable, the relative crack depth, a/t, was further considered (Fig. 1c). Note that the surface crack was assumed to have a rectangular shape (constant-depth), and the value of θ/π was limited up to $\theta/\pi = 0.5$. Note also that, as constant-depth surface cracks are considered, the results in the limiting case of $a/t \to 1$ should recover those for through-wall cracks.

The circumferential crack is assumed to be either in the centre of the elbow or in the junction between the elbow and the attached straight pipe (Fig. 1a). For each location, the crack either at extrados or at intrados was further considered.

2.2. Finite element analysis

Fig. 2 depicts typical FE meshes for elbows with circumferential through-wall and surface cracks in the centre of the elbow. Due to symmetry, only a quarter model was used. The crack-tip was designed with collapsed elements, and a ring of wedge-shaped elements was used in the crack-tip region. For through-wall crack cases, two elements were used through the thickness. It should be noted that mesh sensitivity study using four and six elements through the thickness was also