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# Boundary effect on weight function in nonlocal damage model

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ABSTRACT

Some insights on boundary effects in nonlocal damage modelling are addressed. Interaction stresses that are at the origin of nonlocality are expected to vanish at the boundary of a solid, in the normal direction to this boundary. Existing models do not account for such an effect. We introduce tentative modifications of the classical nonlocal damage model aimed at accounting for this boundary layer effect in a continuum modelling setting. Computations show that some nonnegligible differences may be observed between the classical and modified formulations. In a one dimensional spalling test, only the modified formulation provides a spall of finite nonzero thickness, whereas spalls smaller than the internal length cannot be obtained according to the original formulation. For the same set of model parameters, including the internal length, the fracture energy derived from the size effect test method is also very different according to both approaches. Parameters in the size effect laws for notched and unnotched specimens, obtained from computation of geometrically similar bending beams, are more consistent with the modified nonlocal model compared to the original nonlocal formulation.

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## 1. Introduction

Most failure models for strain softening materials involve nonlocality. Whether nonlocality is introduced in an integral or in a gradient format, an internal length is added to the material description. Such constitutive relations provide consistent continuum failure models for progressive cracking in quasi-brittle materials (see e.g. [3]) or ductile failure in alloys (see e.g. [16]). In quasi-brittle materials at least, nonlocality finds its origin in the interaction between growing defects in the course of failure. When a microcrack opens, stresses are released and the stress field in the neighbourhood of the crack is modified accordingly which may induce some further cracking elsewhere. These interactions may be approximated following the superposition scheme due to Kachanov [14] for instance and folded into micromechanical damage based models (see e.g. Refs. [2,21,22]). There are at least two outcomes from these approaches: first, the weight function that is introduced in the nonlocal averaging, along with the internal length, is recovered; second this weight function depends on the state of damage and it is direction dependent with respect to the state of stress. Cracks may shield each other or amplify the interaction stresses acting in their neighbourhood.

Nearby the boundary of the solid, interactions between defects are expected to be different compared to those observed in the bulk material. Such boundary effects are among the pending issues in nonlocal modelling for which very little is known from an experimental or a theoretical point of view. In nonlocal models, boundaries are usually dealt with arbitrarily: in integral models the weight function involved in the nonlocal average is chopped off and normalized [20]. It follows that

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**Nomenclature**

| | |
|---|---|
| $\boldsymbol{\sigma}_\infty$ | remote stress field in finite solid |
| $\sigma_{ij}$ | stress components |
| $\varepsilon_{kl}$ | strain components |
| $C_{ijkl}$ | components of the fourth-order elastic stiffness tensor |
| $D$ | damage variable |
| $E$ | Young's modulus |
| $\nu$ | Poisson's ratio |
| $\varepsilon_{eq}$ | equivalent strain |
| $\langle\varepsilon_i\rangle_+$ | positive principal strain |
| $D_t$ | tensile damage |
| $D_c$ | compressive damage |
| $\Psi(\boldsymbol{x}-\xi)$, $\Psi_0(\boldsymbol{x}-\xi)$, and $\Psi^*(x,\xi)$ | weight functions |
| $\xi$, $\boldsymbol{x}$ | coordinate system |
| $\bar{\varepsilon}_{eq}$ | nonlocal equivalent strain |
| $\Omega$ | volume of the structure |
| $l_c$ | internal length of the nonlocal continuum |
| $\delta(\boldsymbol{x}-\xi)$ | Dirac delta function |
| $a$ | minimum between the internal length and the distance from the point to the closest boundary |
| $b$ | minimum between the internal length and the distance to the boundary of the solid in the orthogonal direction |
| $A_t$, $A_c$, $B_t$, $B_c$ and $\varepsilon_{D_0}$ | parameters in the evolution law for damage |
| $P_u$ | ultimate load in the size effect tests |
| $\sigma_N$ | nominal strength in size effect tests |
| $D_b$ | parameter in size effect law for unnotched specimen |
| $f_{r\infty}$ | modulus of rupture for specimen of infinite size |
| $B$ and $D_0$ | parameters in size effect law for notched specimens. $B$ depends on the geometry, $D_0$ is a characteristic size |
| $G_f$ | fracture energy computed according to size effect law |

the influence of a point A located nearby a boundary on a point B located in the bulk of the solid is not the same as the influence of B on A. Due to the truncature of the interaction domain and to the renormalization, the weight function centered at point A and entering in the nonlocal averaging at point A is not the same as the weight function centered at point B and entering in the nonlocal averaging at point B. In the context of continuum damage modelling, this peculiarity of integral nonlocal models was pointed out many times (see e.g. [19,4]). It is at the origin of the loss of symmetry of the tangent operator in the nonlocal integral formulations. Because the nonlocal interactions are changing nearby the boundary of the solid, the constitutive formulation [20] does not derive from a thermodynamic free energy potential. The modified symmetric nonlocal damage theory due to Borino and coworkers [6] derives from a potential and fulfill thermodynamic principles. To this end, the weight function is modified near the boundary. The background for such a modification is the symmetry of the nonlocal interactions and energy considerations, it is not related to some specific boundary effect that would arise from the interaction between microcracking and the boundary of the solid.

In gradient enhanced models, the normal component of the gradient of the nonlocal variable is constrained to be zero on the boundary. In fact, the free boundary condition on the nonlocal variable is the same as the condition that would be induced by an axis of symmetry. It means that the nonlocal interactions nearby a boundary of the solid are the same as the nonlocal interactions that would be observed nearby an axis of symmetry. It can be hardly admitted, however, that the interaction between defects and a boundary surface is the same as the interaction between defects distributed symmetrically in a bulk material. Note that in displacement based gradient models [25,11], the displacements ought to be equal to the nonlocal displacements on the boundary of the solid. This is again a different boundary condition.

In any case, there is very little theoretical motivation for such boundary conditions in nonlocal models, at least some justification is lacking. It may be argued that boundary conditions are not very important. Generally, cracks propagate inside the structure and the fracture process zone is located in the bulk material. Initiation of cracking, however, very often occurs from the boundary of a solid. The simplest situation is that of bending beams. It is expected that the boundary effect may have some influence on the initiation condition of cracking and this is the primary motivation for the present study. Once a crack has propagated, it forms a new, evolutive, boundary of the solid and the nonlocal formulation should account for this additional boundary effect. This case is outside the scope of this paper, along with the issue of nonlocal effects nearby interfaces.

Our purpose is to provide some insight on the boundary effects induced by nonlocality and therefore to investigate the effect of subsequent modifications of nonlocal averaging nearby boundaries in integral damage models. Let us start with some intuitive argument about nonlocality nearby the boundary of a solid [23]. Consider a finite body that contains a population of microcracks or microvoids in an elastic matrix. Given a set of boundary conditions, the mechanical response of this body may be described following two techniques: in the first one, the elastic material containing the defects (cracks and voids) is homogenised. The result is a constitutive relation at each material point that depends on the defect density and

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