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Note

Three-dimensional stress intensity factor calibration for a stiffened cracked plate

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A B S T R A C T

Three-dimensional finite element analyses are used in this paper to calibrate the stress intensity factor in a cracked stiffened plate subjected to remote uniform traction. An accurate numerical determination of the stress field and stress intensity variation through the thickness of a central cracked plate was first carried out in order to evaluate three-dimensional effects. A stiffened cracked plate was then analysed, taking into account the results and the conclusions obtained in the previous study. Such a structure was chosen due to the growing interest for large integral metallic structures for aircraft applications, following the continuous need for low cost and the emergence of new technologies. The *J*-Integral technique was used to calculate the values of the stress intensity factor along the plate thickness. The plane strain behaviour near the crack front and the variation of the opening stress are discussed.

1. Introduction

An aircraft fuselage structure includes, among other parts, the external skin and longitudinal stiffeners (stringers and longerons) eg. [1]. Since the 1920s, investigations on the strength and behaviour of aluminium alloy stiffened sheet specimens used in aircraft construction were carried out [2,3]. Stiffened panels are light and highly resistant sheets reinforced by stringers designed to cope with a variety of loading conditions. Stiffeners improve the strength and stability of the structure and provide a mean of slowing down or arresting the growth of cracks in a panel. Most common stiffener cross-sections are bulb, flat bar or T- and L-sections, that can be bonded, extruded, connected by means of fasteners, machined or welded to form a panel.

Riveted and bolted stiffeners tend to remain intact as the crack propagates beneath them, providing an alternative path for the panel load to pass. Also, riveted stiffeners continue to limit crack growth after the crack propagation over the stiffener, since a crack cannot propagate directly into the stiffener. The continuous need for low cost and the emergence of new technologies has brought interest for large integral metallic structures for aircraft applications. Evaluation programs for replacement of traditional fastening with these new emerging technologies have been carried out all over the aircraft sector, e.g. [4]. Studies show that in an integral stiffener (machined, extruded or welded) a crack propagates simultaneously in the stiffener and in the skin beneath the stiffener at approximately the same rate. In this case, the crack may propagate into the stiffener and break it [5]. Also, it was observed that the rate of crack growth is significantly reduced in the presence of stiffeners [6].

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Nomenclature

2D	two-dimensional
3D	three-dimensional
COD	crack opening displacement
DBEM	dual boundary element method
FEM	finite element method
SIF	stress intensity factor
SST	singularity subtraction technique
b	half plate width
c	half crack length
$D_{p\varepsilon}$	degree of plain strain
h	half plate height
K	stress intensity factor
K_{2D}	two-dimensional stress intensity factor
K_{3D}	three-dimensional stress intensity factor
$K_{average}$	average value of stress intensity factor
KI	mode I stress intensity factor
t	plate thickness
δ	plane strain zone size
σ_x	stress in the *xx* direction
σ_y	opening stress
σ_z	stress in the thickness direction
σ	remote stress
ν	Poisson's ratio

Linear elastic fracture mechanics in conjunction with the Paris law [7] are widely used to analyze and predict crack growth and fracture behaviour of aircraft panels. During the past decades, several studies have been conducted to calculate stress intensity factors in cracked stiffened panels. Configurations involving cracks in infinite and semi-infinite plates with integral or discretely attached stiffeners have been studied by several authors, e.g. [8]. A compilation of results is presented in a parametric form in [9]. It was noticed that the stress intensity factor (SIF) decreases as the crack approaches a stiffener, indicating that the stiffener aided in restraining the crack or slowing down the propagation. Experiments on box girders with welded stiffeners to study crack growth aspects and remaining life prediction concluded that rigorous finite element analysis has to be performed to compute SIF for a structural component having a complex geometry [10]. In this case, it is important to take into account the effect of nearby boundaries by using numerical methods of structural analysis.

Stress intensity factors for riveted stiffened cracked panels were calculated using the finite element method (FEM) in conjunction with strain energy release rate and the crack tip opening displacement [11,12]. The use of special crack tip enriched elements to solve problems involving edge cracks in stiffened panels was reported in [13]. The application of the complex variable method combined with compatible deformations to finite stiffened panels, by using boundary collocation method was extended in [14].

The purpose of this work is to obtain three-dimensional (3D) stress intensity factor solutions for a cracked stiffened plate using the FEM. In order to check the 3D solutions, results previously obtained by the authors using the compounding method were used [15]. In the compounding method, a complex cracked structure containing n boundaries is separated into a number of simpler ancillary configurations, each of these containing usually only one boundary that interacts with the crack, and has known SIF solutions from handbooks. The stress intensity factor for the complex geometry is obtained by compounding these solutions according to the general equation:

$$K = K_0 + \sum_{i=1}^{n}(K_i - K_0) \tag{1}$$

where K_i is the stress intensity factor for the cracked structure having only the ith boundary and K_0 is the stress intensity factor in the absence of all boundaries. This method was developed by Cartwright and Rooke [16]. Extensive descriptions of the method and different examples may can be found also in [17] and [18].

The numerical investigations of cracked structures feature both two-dimensional (2D) and 3D analyses. The 2D ones are simpler and, in most cases, have a reasonable degree of accuracy. Since the stress state near the crack tip is always 3D, this complex analysis of cracked structures has been used extensively in the last years, in order to produce more accurate numerical solutions. Three-dimensional analyses of thin cracked plates were presented in [19,20], where, in order to obtain stress distributions and stress intensity factor values, refined 3D finite element analyses were performed.

Kwon and Sun presented refined 3D analyses where the stress field near the crack tip, the degree of plane strain and the crack tip singularity were investigated [21]. They suggested also a simple technique to determine 3D stress intensity factor at the mid-plane of a thin plate from a 2D analysis.