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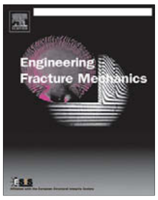

A new method for determining mode II R-curve by the End-Notched Flexure test

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A new method for obtaining the mode II R-curve in a End-Notched Flexure test is proposed in the present work. New compliance and energy release rate equations have been derived incorporating shear, local deformation and bending rotation effects.

Mode II R-curve, which represents energy release rate as a function of crack extension, is obtained without optical determination of crack tip position. Crack length and energy release rate are determined at each point of the test based on experimental compliance until unstable advance occurs. In order to confirm the theoretical models, unidirectional carbon/epoxy specimens have been tested. Experimental data are evaluated by means of two reduction schemes: an existing data method named Corrected Beam Theory with effective crack length and the new method named Beam Theory including Bending Rotation effects. Shear and local deformation effects are included in both reduction schemes.

Results concerning the determination of crack length without crack advance and during stable crack propagation are presented. The agreement between experimental values and theoretical results obtained by the new approach is excellent. Based on the accurate crack length determination at each point of the test, energy release rate is determined point to point and therefore R-curve is obtained.

1. Introduction

Delamination is one of the most common failure modes in composite materials. Mode I delamination test methods have progressed to an international standard [1,2] but there has been a little progress in mode II standard. The resistance of composites to delamination can be well characterized by the fracture toughness, measured as the energy dissipated per unit area of crack growth named energy release rate G. Resistance curves (R-curve) are used in order to evaluate the fracture behaviour of composite materials [3,4].Three mode II tests are usually employed: the *End-loaded split* (ELS), the *Four-point bend End-Notched Flexure test* (4-ENF) and the *Three-point bend End-Notched Flexure test* (ENF) [5–11].

ENF test has been widely used for the determination of the mode II delamination toughness of laminated composites. The main advantage of this test is that it is carried out by a simple three-point bending test. Nevertheless, a drawback in the use of ENF is that crack propagation is inherently unstable. The initial crack length must be at least 0.7 times the half span length in order to the crack propagation being stable [12,13]. Several authors have also reported that ENF test allows only the determination of initial values but not of resistance curves [14,15].

One of the limiting factors of this test method is the precise determination of the crack length. Russell and Street [16] developed a solution based on a simple beam theory neglecting transverse shear deformation and crack tip singularity.

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Nomenclature

BTBR	Beam Theory including Bending Rotations
MCBTE	Modified Corrected Beam Theory with Effective length including shear and local deformations effects
M_1, M_2, Q_1, Q_2	moments and shear forces in the cracked zone
P	applied load
$P_{\max}$	maximun load
Y	internal force in the cracked zone
R_{A}	reaction at support A
I	second moments of area with respect to the middle plane of the lower and upper beam in the cracked zone
A	surface areas of the lower and upper beam in the cracked zone
h_1, h_2	thicknesses of the lower and upper beam in the cracked zone
χ	correction factor of the energy release rate
$2L_0$	span of the undeformed configuration
d, $(2L)_{\mathrm{L}}$	dimensions of the deformed configuration
w, $2h$	width and thickness of the specimen, respectively
R_1, R_2, R_3	radius at support and loading noses
E_{f}	flexural modulus
G_{LT}	shear modulus
C_{S}, k	local deformation coefficients
$\delta_{\mathrm{M}}, \delta_{\mathrm{Q}}$	displacements contributions corresponding to bending and shear, respectively
δ_{Exp}	experimental displacement
δ_T	total displacement
δ	displacement of the specimen
δ_0	displacement correction
$\eta_{\mathrm{A}}, \eta_{\mathrm{B}}, \eta_{\mathrm{C}}$	terms related to support and load span reduction
$\theta_{\mathrm{A}}, \theta_{\mathrm{B}}, \theta_{\mathrm{C}}$	bending angles
$\alpha_a, \alpha_f, \alpha_s$	correction factors due to bending rotations
ε	strain of the middle section of the specimen
$a^{\mathrm{II}}, a^{\mathrm{I}}$	crack length taking and without taking into account bending rotation effects, respectively
a_e	crack length in the deformed configuration
a_0	normalized crack length
a_r	actual crack length
$a^{\mathrm{II}}_{\mathrm{m}}, a^{\mathrm{I}}_{\mathrm{m}}$	mean values of the crack length taking and without taking into account bending rotation effects, respectively
$a^{\mathrm{II}}_{\mathrm{lp}}, a^{\mathrm{I}}_{\mathrm{lp}}$	crack length at the last point of the test taking and without taking into account bending rotation effects, respectively
$\Delta a^{II}, \Delta a^{I}$	crack extension taking and without taking into account bending rotation effects, respectively
$C^{\mathrm{II}}, C^{\mathrm{I}}$	compliances taking and without taking into account bending rotation effects, respectively
$C^{\mathrm{II}}_{\mathrm{m}}, C^{\mathrm{I}}_{\mathrm{m}}$	compliances for the initial crack length taking and without taking into account bending rotation effects, respectively
C	compliance of the specimen
$G^{\mathrm{II}}_{\mathrm{II}}, C^{\mathrm{I}}_{\mathrm{II}}$	energy release rate taking and without taking into account bending rotation effects, respectively

Carlsson et al. [12] found that interlaminar shear deformation may influence the evaluation of the interlaminar mode II fracture toughness. The influence of friction effects was also analysed. Corleto and Hogan [17] presented a modified beam theory solution of the ENF that incorporates the effect of crack tip deformation on the mode II critical energy released rate G_{IIc} using the solution of a beam on a generalized elastic foundation.

Wang and Williams [18] developed a new method to correct fracture toughness obtained by the ELS and ENF. Ding and Kortschot [19] presented a modified classical beam theory solution of the ENF specimen without considering transverse shear deformation. Wang and Qiao [20] developed a novel beam model of the ENF test. The specimen was modelled as two sub-problems using the principle of superposition, obtaining an equation of G_{IIc} similar to Wang and Williams.

Szekrényes [21] presented an improved analysis for unidirectional composite delamination specimens including four mechanical deformations apart from simple beam theory: Winkler–Pasternak foundation, transverse shear, Saint–Venant effect and crack tip shear deformation, respectively.

The inconvenience of the referred methods is the necessity of crack measuring during propagation, which is experimentally difficult since the crack grows without a clear opening. To overcome this problem Tanaka et al. [22] proposed a method for evaluating crack length in ENF test by Crack Shear Displacement (CSD) control. The CSD is the relative shear slip between upper and lower crack surfaces of the ENF specimen, which is measured using a special displacement gauge. Although their method can determine the crack length without observation, a servo valve-controlled testing machine is required. Thus, the testing procedure is more complicated than the one based on loading point displacement.