Engineering Fracture Mechanics 76 (2009) 876–883

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

# The shielding effect of the imperfect interface on a mode III permeable crack in a layered piezoelectric sensor

Yong-Dong Li [a,b], Kang Yong Lee [b,*]

[a] Department of Mechanical Engineering, Academy of Armored Force Engineering, No. 21, Du Jia Kan, Chang Xin Dian, Beijing 100072, PR China
[b] School of Mechanical Engineering, Yonsei University, Seoul 120-749, Republic of Korea

ARTICLE INFO

*Article history:*
Received 20 August 2008
Received in revised form 12 December 2008
Accepted 16 December 2008
Available online 25 December 2008

*Keywords:*
Piezoelectric sensor
Imperfect interface
Permeable crack
Shielding effect
Stress intensity factor

ABSTRACT

The mechanical model is established for a piezoelectric sensor with a mode III permeable crack parallel to the imperfect interface. Fracture analysis is performed by the standard methods of Fourier transform and singular integral equation. Three conclusions are drawn: (a) the imperfect interface has a shielding effect on the crack parallel and very near to it; (b) the shielding effect depends on the structural stiffness and the distance between the crack and interface; (c) for the electrically permeable crack, mechanical imperfection has more remarkable shielding effect than dielectric imperfection does.



## 1. Introduction

Sensors are widely used in many kinds of modern smart devices, and the kernel components of some advanced sensors are piezoelectric ceramics. Most piezoelectric sensors available in engineering are layered structures, and the simplest ones are typically composed of a piezoelectric layer and a dielectric substrate bonded together by a thin interphase of epoxyn. Because the thickness of the interphase is so small that it is always simplified in mechanical analysis as an idealized interface with no thickness. Under the action of electro-mechanical loading, the interfaces might be damaged. As a result, the original perfectly bonded interfaces become imperfect, i.e., some pertinent electro-mechanical quantities become discontinuous across the interfaces [1]. In order to simulate the damage of the interface, several interface models have been established in the existing literatures, e.g., the spring-type model [2], the coherent model [3] and etc. In these models, the most widely used one is the spring-type model, which is effective in simulating the electro-mechanical imperfection of the piezoelectric interface. Many researchers have employed the spring-type model to study the mechanical problems of piezoelectric structures, for example, the vibration of a piezoelectric laminated cylinder [4], the bending of angle-ply piezoelectric laminates [1,2], the uniform tension of a piezoelectric fiber composite [5], the elastic waves in bonded piezoelectric materials [6,7], the piezoelectric screw dislocations interacting with an imperfect interface [8–10], and so on.

In engineering, piezoelectric ceramics are prone to fracture during in-situ services due to their intrinsic brittleness. Therefore, fracture analysis is a key problem in the design and application of piezoelectric devices. In this field, how to impose electrical boundary conditions on crack surfaces and how to choose the fracture parameter for piezoelectric materials still remain two controversial problems. For the former problem, there are two completely opposite opinions. One is the permeable opinion [11] and the other the impermeable one [12]. Besides these two kinds of opinions, the model of

* Corresponding author. Tel./fax: +82 2 2123 2813.
*E-mail addresses:* lydbeijing@163.com (Y.-D. Li), kyl2813@yonsei.ac.kr (K.Y. Lee).

0013-7944/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engfracmech.2008.12.011

**Nomenclature**

$\alpha_1$ and $\alpha_2$ mechanical and dielectric imperfection parameters of the interface
$w$, $\tau$, $\varphi$ and $D$ anti-plane mechanical displacement, stress, in-plane electric potential and electric displacement
$c_{44}$, $e_{15}$ and $\varepsilon_{11}$ shear modulus, piezoelectric coefficient and dielectric coefficient
$E_0$ and $\tau_0$ applied electric field and anti-plane traction
$h_l$ and $h_s$ thickness of the piezoelectric layer and elastic substrate
$a$ half-length of crack
$g(x)$ unknown auxiliary function
$K_\tau$, $K_D$, $K_\gamma$, $K_E$ and $G$ stress intensity factor, electric displacement intensity factor, strain intensity factor, electric field intensity factor and energy release rate
$\widetilde{K}_\tau, \tilde{\alpha}_1$ and $\tilde{\alpha}_2$ normalized stress intensity factor and interface parameters

limited-permeable crack has also been proposed [13]. For the latter problem, several different fracture parameters have been put forth, including the intensity factors, the total energy release rate, the mechanical strain energy release rate and the energy density factor.

For a cracked piezoelectric sensor with an imperfect interface, the effect of the imperfect interface on the crack is a problem of practical significance, which may contain implications for the anti-failure design of piezoelectric sensors. Up till now, investigations on such an effect have not been reported in the existing literatures, to the best of our knowledge. The present work aims at studying this effect. The mechanical model is established for a piezoelectric sensor with a mode III permeable crack parallel to the mechanically compliant and weakly conducting interface. Fracture analysis is performed by the standard methods of Fourier integral transform and Cauchy singular integral equation. Stress intensity factor is chosen as the fracture parameter, and the effect is discussed of the imperfect interface on the fracture responses of the piezoelectric sensor.

## 2. Problem formulation

Illustrated in Fig. 1 is a sensor composed of a piezoelectric layer and a dielectric substrate bonded through a soft interphase, which is so thin as to be modeled as an interface with no thickness. In practice, the interphase is sometimes damaged mechanically and/or dielectrically under harsh conditions. It is found that [7–9], across an imperfect interphase, the normal tractions and normal electric displacement remain continuous, but the mechanical displacement and/or the electric potential may become discontinuous. In linear cases, the jumps of the mechanical displacement and the electric potential are proportional to the pertinent stress and electric displacement components, respectively. Therefore, the mechanical and dielectric imperfections of the interface can be described by two "spring-type" parameters. For simplicity, we only consider the anti-plane fracture problem here, and the imperfect interface are characterized as

$$\boldsymbol{\tau}_{yl}(x,0) = \boldsymbol{\tau}_{ys}(x,0) \tag{1}$$

$$\mathbf{w}_l(x,0) - \mathbf{w}_s(x,0) = \boldsymbol{\alpha}\boldsymbol{\tau}_{yl}(x,0) \tag{2}$$

where $\boldsymbol{\tau}_k = [\tau_{kz}, D_k]^{\mathrm{T}}$ ($k = x,y$) and $\mathbf{w} = [w,\varphi]^{\mathrm{T}}$. $w$, $\tau$, $\varphi$ and $D$ are anti-plane mechanical displacement, stress, in-plane electric potential and electric displacement. The subscripts $l$ and $s$ refer to the quantities of the layer and substrate, respectively. $\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 & 0 \\ 0 & -\alpha_2 \end{bmatrix}$. $\alpha_1$ and $\alpha_2$ are the mechanical and dielectric imperfection parameters of the interface [7–9]. It deserves noting that the test for these two parameters is of practical significance for applications of piezoelectric devices. Unfortunately, the testing results have been scarcely reported and the two interface parameters are not available in practice now. The measurement for them still deserves studying.

The piezoelectric layer is poled along the $z$ direction. Under the condition of anti-plane strain, the constitutive relations of the dielectric substrate and transversely isotropic piezoelectric layer are

$$\boldsymbol{\tau}_{kj} = \mathbf{M}_j\mathbf{w}_{j,k}, (j = l,s;\ k = x,y) \tag{3}$$

where and hereafter the indices following a comma denote partial differentiations and the Einstein summation convention is not applied. The property matrices are

$$\mathbf{M}_j = \begin{bmatrix} c_{j44} & \delta_{jl}e_{l15} \\ \delta_{jl}e_{l15} & -\varepsilon_{j11} \end{bmatrix}, \quad (j = l,s) \tag{4}$$

where $\delta_{jl}$ is the Kronecker delta, which is 1 if its two subscripts are identical and 0 otherwise. $c_{44}$, $e_{15}$ and $\varepsilon_{11}$ are the shear modulus, piezoelectric coefficient and dielectric coefficient.

When body forces and free charges are neglected, the governing equations are

$$\mathbf{M}_j\nabla^2\mathbf{w}_j = \mathbf{0}, \quad (j = l,s) \tag{5}$$

where $\nabla^2$ is the Laplace operator.