Engineering Fracture Mechanics 78 (2011) 1565–1576

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Analysis of the crack growth propagation process under mixed-mode loading

Slobodanka Boljanović *, Stevan Maksimović

VTI-Aeronautical Department, Ratka Resanovića 1, 11000 Belgrade, Serbia

ARTICLE INFO

Article history:
Available online 4 March 2011

Keywords:
Mode I/II conditions
Crack path
Fatigue life estimation
Numerical modeling

ABSTRACT

In the present paper, a computational model for crack growth analysis under Mode I/II conditions is formulated. The focus is on two issues – crack path simulation and fatigue life estimation. The finite element method is used together with the maximum principal stress criterion and the crack growth rate equation based on the equivalent stress intensity factor. To determine the mixed-mode stress intensity factors, quarter-point (Q-P) singular finite elements are employed. For verification purposes, a plate with crack emanating from the edge of a hole is examined. The crack path of the plate made of 2024 T3 Al Alloy is investigated experimentally and simulated by using the finite element method with the maximum tangential stress criterion. Then, the validation of the procedure is illustrated by applying the numerical evaluation of the curvilinear crack propagation in the polymethyl methacrylate (PMMA) beam and the Arcan specimen made of Al Alloy for which experimental results are available in the literature. In order to estimate fatigue life up to failure of the plate with crack emanating from the edge of a hole, the polynomial expression is evaluated for the equivalent stress intensity factor using values of stress intensity factors obtained from the finite element analysis. Additionally, the fatigue life up to failure of the Arcan specimen is analyzed for different loading angles and compared with experimental data. Excellent correlations between the computed and experimental results are obtained.

1. Introduction

The engineering practice experience shows that cracks which initiate in positions with microstructure in-homogeneities, welding or casting defects as well as defects which initiate by the hostile environment lead to extension of cracks during service life, and cause the reduced loading capacity of the structural components. Such dangerous cracks generally have arbitrary orientations and must be analyzed with a special attention. Fatigue problems that occur due to arbitrarily oriented cracks are usually referred to either the multi-axial fatigue problem [1–6] or the mixed-mode fatigue problem [7–13] within the context of fracture mechanics.

The crack growth analysis under mixed-mode loading has had a long standing interest in theory and experiment. Theoretical considerations enable description of practical problems of fracture by different principles of fracture mechanics. In order to study the crack growth process which occurs under mixed-mode loading, theoretical considerations must include both crack path direction and life estimation. Moreover, from the engineering point of view, suggested criteria and laws

* Corresponding author. Tel.: +381 63 805 60 85; fax: +381 11 35 11 282.
E-mail addresses: slobodanka.boljanovic@gmail.com (S. Boljanović), s.maksimovic@open.telekom.rs (S. Maksimović).

0013-7944/$ - see front matter
doi:10.1016/j.engfracmech.2011.02.003

Nomenclature

a	crack length
a_f	final crack length
a_0	initial crack length
C	Paris equation constant
da/dN	crack growth rate
E	elastic modulus
K	stress intensity factor
L	specimen length
m	Paris equation exponent
M^e	elastic mixity parameter
N	number of loading cycles up to failure
P	external load/force
R	load/stress ratio
t	specimen thickness
w	specimen width
x_c, y_c	Cartesian co-ordinates at the crack tip
β	constant
φ	loading angle
ν	Poisson's ratio
θ	angle of crack extension

Subscripts

eq	equivalent
max	maximum value (of a given load)
r, θ	polar co-ordinates at the crack tip
I, II	mode of loading

defined in theoretical investigations could be used for formulating a computational model for the crack growth analysis under mixed-mode loadings with high quality calculations.

Theoretical criteria can be categorized in three groups based on the definition of crack growth direction under mixed-mode loadings. The first established criterion is the Griffith's criterion based on the maximum strain energy release rate, which was later modified and improved by various authors, such as Hussain et al. [14], Hyashi and Nemat-Nasser [15], Ueda et al. [16]. Hussain et al. introduced the G-criterion in which it was assumed that the crack initiation occurs in the direction with maximum strain energy release rate. In the second group, there is a criterion proposed by Erdogan and Sih [17], known as the MTS criterion. According to this criterion, fracture occurs in the direction where the tangential stress is maximum. Additionally, as a result of investigations that followed, some authors suggested modifications: for example, Smith et al. [18] took in consideration T-stress and developed a generalized MTS criterion. Furthermore, Sih [19] developed the minimum elastic strain energy criterion (S-criterion). The S-criterion, as a representative of the third group of criteria, assumed that a crack extension started in the initiation direction when strain energy density reached a critical value. Theocaris and Andrianopoulos [20] modified the S-criterion and suggested the T-criterion. In the T-criterion, it was defined that, along the contour of constant effective stress, the crack initiation occurs in the direction with maximum dilatational strain energy density.

The criteria to be applied in practice should be simple to use, contain a small number of input parameters, and enable satisfactory evaluations. So far, investigations have shown that application of the MTS criterion in the crack growth analysis provides good estimations. Therefore, Selcuk et al. [21] employed the MTS criterion for their study of interfacial crack path using a direct boundary integral approach. Subsequently, Plank and Kuhn [22] have also used the MTS criterion for their analysis of non-proportional mixed mode problems.

Moreover, in the literature, there is a number of different crack growth laws for evaluation of fatigue crack growth rate and life estimation under mixed-mode loading. All mixed mode crack growth laws for life estimations employ different fracture mechanics parameters in order to take into account Mode I/II conditions. Thus, as far as mixed mode problems are concerned, Tanaka [7] has modified the Paris law by defining the equivalent stress intensity factor. Thereafter, Socie et al. [8], Yan [9] and later Tong et al. [10] suggested their relationships for the equivalent stress intensity factor as a function of the stress intensity factors for Mode I and Mode II. Richard [11] introduced an equation for the equivalent stress intensity factor in which a material feature (fracture toughness) and the stress intensity factors for Mode I and Mode II are considered. Furthermore, Hoshide and Socie [12], and later Chen and Keer [13] proposed their relations for crack growth rate using *J*-integral for mixed mode situations.

A very important remark in the context of mixed mode problems is that existing criteria and laws based on fracture mechanics cannot describe well the local plasticity around the crack tip. This deficiency could be minimized by introducing