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In-plane and out-of-plane crack-tip constraint effects under biaxial nonlinear deformation

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ABSTRACT

In-plane and out-of-plane constraint effects on crack-front stress fields under both elastic–plastic and creep conditions are studied by means of three-dimensional numerical analyses of finite thickness boundary layer models and plane strain reference solutions. This investigation is an extension of the plane strain solution obtained by Shlyannikov and Boychenko in 2008, with special attention on what constraint parameters existed in the nonlinear crack-tip fields in a finite thickness solid. Characterization of constraint effects is given by using the non-singular *T*-stress, the local triaxiality parameter, the factor of the stress-state in 3D cracked body and the second order term amplitude factor. The influence of nominal stress load biaxiality and creep time on the behavior of constraint factors is considered. Stresses and constraint factors from FEA at the crack-front on different planes in the thickness direction of the plate are compared with plane strain reference solutions. The results show that 3D-stress fields can be characterized in common with the local triaxiality parameter and factor of the stress-state in 3D solid by the three-term solution throughout the thickness even in the region near the free surface. It is found that there is a distinct relationship between the in-plane and the crack-front out-of-plane constraint factors which can be well captured using the relation between the second order term amplitude factor and remote boundary layer stress.

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1. Introduction

Constraint effects in modern fracture mechanics are usually considered as specimen configuration and loading conditions influence on crack-tip fields. Therefore, fracture toughness dependence is referred to these factors and cannot be used as a constant of material. However, the general discussion of constraint effects requires to be defined more exactly. Constraint effects can be defined as specimen prevention from plastic strains depending on geometry and loading conditions.

Constraint effects near the crack tip have been extensively studied for a long time. Most of the researches are referred to in-plane constraint since Williams [1] presented the asymptotic expansion of the stress field around the crack tip in elastic body that includes a non-singular in-plane normal stress component, the *T*-stress. Subsequently Larsson and Carlsson [2], and Rice [3] have shown that including the *T*-stress gave an improved representation of the elastic–plastic crack tip stress fields. Based on *T*-stress, a two-parameter approach *J*–*T* was proposed by Betegon and Hancock [4], which takes into account the in-plane constraint on crack-tip fields. O'Dowd and Shih [5] have introduced as alternative constraint methodology a two-parametrical approach on the base of *J* and a hydrostatic stress parameter *Q*. The J–A_2 two-parameter three-term approach was also proposed to describe the stress field in the vicinity of the crack tip in a power hardening material [6,7]. Shlyannikov and Boychenko [8] have discussed the influence of biaxial loading on the higher-order both stress and strain rate

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Nomenclature

a	crack length
A_1, A_2, A_3	higher-order term amplitude factors
B	inherent biaxiality parameter
$\bar{B}$	creep parameter
$C(t)$	creep amplitude factor
E	the Young's modulus
$f_{ij}(\theta)$	dimensionless elastic angular stress functions
h	local stress triaxiality parameter
K	elastic stress intensity factor
m	strain hardening exponent
n	creep exponent
$\bar{r}$	normalized crack tip distance
r,θ	polar coordinates centered at the crack tip
s_1, s_2, s_3	exponent of stress functions
t	creep time
t_T	time for transition from small-scale creep to extensive creep
T	local non-singular stress
T_{appl}	remote boundary layer stress
T_z	factor of the stress-state in 3D cracked body
u_x, u_y	displacement components
β	crack angle
σ	nominal stress in the Y-axis direction
σ_{ij}	stress tensor components
σ_y	the yield stress
$\tilde{\sigma}_{ij}^{(k)}(\theta)$	dimensionless elastic–plastic angular functions
$\tilde{\sigma}_e$	the Mises effective dimensionless stress
σ_0	reference stress
$\dot{\varepsilon}_0$	reference creep strain rate
η	nominal stresses biaxial ratio
ν	the Poisson's ratio

fields and the second order amplitude coefficients for idealized plane strain conditions. However, all these approaches can successfully describe the in-plane constraint effects, but they are limited to a planar case. The description of out-of-plane constraint should include specimen's dimension such as thickness. Only a few researches have been done to describe thickness effect on the crack-tip constraint [9–12]. Some authors [13,14] showed that 3D crack-front constraint effect in a thin plate and in thick SENB specimens are well represented by J–A_2 three-term solution under small-scale yielding and large-scale yielding conditions.

This study focuses on the finite element analysis of the plastic and creeping materials under different in-plane and out-of-plane constraint levels. The geometry considered in detailed three-dimensional finite element calculations is a biaxially loaded finite thickness plate. Different degree of in-plane constraint is given in form of the applied stress T_{appl} by combination of far-field stress level, biaxial stress ratio and initial crack angle. In our case the T_{appl} is a parameter describing solely loading conditions. Loadings and initial crack angle were applied related to a range $(-1, +1)$ of far-field biaxial stress ratio. For the purpose of comparison, full-field finite element analysis based on a modified layer approach wherein the T-stress is prescribed as remote boundary conditions is employed to model the effects of biaxial loading on nonlinear behavior under plane strain conditions. The present work is concerned with the application of triaxiality parameter h, T_z-factor and the second order term amplitude A_2 to 3D crack-front stress field and the characterization of the interaction of in and out-of-plane constraint.

2. Determination of constraint factors

It is well known that different traditional approaches, which successfully describe the in-plane constrain, are not accurate for 3D cracks. Thus, it is necessary to use others factors to describe the out-of-plane constraint. The T_z factor introduced by Guo [9] is an important parameter to characterize the constraint effect accurately, which is essential to establish a three parameter dominated stress field, and offers a possibility to characterize the stress-state in a 3D cracked body

$$T_z = \frac{\sigma_{zz}}{\nu(\sigma_{xx} + \sigma_{yy})}, \tag{1}$$

where ν is the Poisson's ratio, and $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ are the stress tensor components.