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A general 3D approach for the analysis of multi-axial fracture behavior of reinforced concrete elements

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ABSTRACT

The behavior of cracked reinforced concrete structural components is here analyzed through a three-dimensional model, which includes all the interface phenomena generated along cracks, such as aggregate bridging and interlock, tension stiffening and dowel action, as well as the non-linear response of concrete in compression and in tension. The model is able to effectively describe the progressive development of multi-axial cracking, by considering the crack re-orientation and the change of the crack spacing as loading increases. The proposed formulation, which is expressed in terms of secant stiffness matrix, is obtained by taking into account the flexibility contributions of cracks and of the concrete between adjacent cracks, in both the singly and the multi-cracked stage. Finally, this model is implemented into a finite element code and is validated through comparisons with significant experimental data available in the literature.

1. Introduction

In reinforced concrete structural members, immediately after the appearance of first (primary) cracking, the crack pattern orientation mainly depends on the stress and strain fields of the uncracked phase, as well as on the spacing and arrangement of reinforcing bars. Subsequently, material discontinuities due to cracks cause a marked change of stress and strain fields in concrete and in reinforcing steel with respect to that observed in the previous uncracked phase [1–4]. Normal and shear forces are transferred through cracks by complex mechanisms, mainly represented by the interface actions at crack surfaces (cohesion, bridging and aggregate interlock) and the interaction effects between concrete and reinforcing bars (dowel action, tension stiffening, bond). These phenomena depend on many parameters such as the shape and quality of aggregates, the arrangement and dimensions of steel bars, the quality of concrete–steel bond and the possible presence of fibers [5–8]. They have been experimentally investigated and reliable mathematical models, available in the literature [9–11], have been proposed in order to evaluate the interface forces as a function of crack opening and slip. Afterwards, when the loading varies – especially if not monotonically – new (secondary) cracks, oriented along different directions with respect to primary cracks (Fig. 1a, [12]), can form with a smaller spacing. Also in this case, the re-orientation of cracks is strictly connected to the anisotropic behavior of RC, which is in turn related to the adopted reinforcement, often characterized by markedly different steel ratios arranged along perpendicular directions (Fig. 1b, [12]).

In this study, a three-dimensional numerical model for the analysis of cracked RC structural components subjected to a multi-axial state of stress is proposed. The model, named 3D-PARC [13,14], represents an extension of the previous 2D-PARC [15,16] and is formulated in terms of secant stiffness matrix to allow its implementation into a FE procedure. In the uncracked stage, concrete behavior is described through a non-linear elastic model, where the elastic moduli are evaluated

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Nomenclature

a_{mk} kth crack spacing
A_{si} cross-sectional area of the ith steel reinforcement layer
$[D]$, $[D_c]$, $[D_s]$ reinforced concrete, concrete and steel stiffness matrices, respectively, evaluated in the x–y–z coordinate system
$[D_c^{123}]$ concrete stiffness matrix in the 1–2–3 coordinate system (being 1–2–3 the principal strain directions)
$[D_{crk}]$, $[D_{c,crk}]$, $[D_{s,crk}]$ kth crack stiffness matrix, stiffness matrices due to concrete and steel contributions in the kth crack, evaluated in the x–y–z coordinate system
$[D_{c,crk}^{n_k t_k}]$ stiffness matrix due to concrete contributions in the kth crack, evaluated in the crack coordinate system n_k–t_k
$[D_{si,crk}^{x_i y_i}]$ stiffness matrix of the ith steel layer crossing the kth crack, in the local x_i–y_i coordinate system of the bar
E_{ci}, ν_i secant values of the longitudinal elastic moduli and Poisson coefficients for concrete in the ith direction (i = 1, 3)
$\overline{E}_{sicrk}$ secant elastic modulus of the ith steel bar at the kth crack
f_c concrete uniaxial compressive strength
f_t concrete uniaxial tensile strength
g_{ik} tension stiffening coefficient
$[I]$ identity matrix
l_{si} length of ith bar between two cracks
u_k, v_k opening and slip of the kth crack
$\{\varepsilon\}, \{\varepsilon_c\}, \{\varepsilon_s\}$ total strain, concrete and steel strain respectively, evaluated in the x–y–z coordinate system
$\{\varepsilon_{crk}\}, \{\varepsilon_{crk}^{n_k t_k}\}$ kth crack strain, evaluated in the global (x–y–z) and in the local (n_k–t_k) crack coordinate systems, respectively
ε_{sicrk} steel strain at the kth crack
ϕ_i diameter of the ith steel reinforcement layer
ρ_{si} geometric steel ratio for the ith reinforcement layer
$\{\sigma\}, \{\sigma_c\}, \{\sigma_s\}$ applied stress state, stress state in uncracked concrete or in concrete between two adjacent cracks, stress state in the steel embedded in uncracked concrete or in concrete between two adjacent cracks, respectively
$\{\sigma_{crk}\}, \{\sigma_{c,crk}\}, \{\sigma_{s,crk}\}$ stresses in the kth crack, stresses in the crack balanced by resistant contributes due to concrete and due to steel bars crossing the kth crack, respectively
σ_{bk}, σ_{ak}, τ_{ak}, τ_{ik} normal stresses due to aggregate bridging effect, normal and shear stresses due to aggregate interlock, shear stresses due to dowel action

through equivalent non-linear uniaxial relationships and the maximum stresses are limited by an appropriate strength domain [13,17]. Singly and multi-cracked stages are simulated by adding crack contributions to the flexibility matrix of the uncracked material, whose properties are properly degraded. These contributions are evaluated through a local analysis of the reinforced concrete component, on the basis of a strain decomposition procedure. The model assumes fixed, multi-directional cracking and smeared reinforcement approaches, taking into account the effects of a multi-axial stress state and several post-cracking phenomena, such as tension stiffening, dowel action and aggregate interlock. In order to verify the reliability of the proposed approach, it is implemented into a FE code and validated through comparisons with significant experimental data.

2. Modeling of RC behavior in the cracked stage

The proposed model is structured in a modular framework. All the mechanical phenomena are individually analyzed on the basis of their properties and physical conditions, and the stiffness matrix is computed separately for each resistant contribution by using suitable constitutive models and techniques.

2.1. Aggregate bridging and interlock contributions

After crack formation in RC structural components, crack opening and slip activate several resistant mechanisms, due to concrete and reinforcing steel. The first contributions, which are mainly due to aggregates crossing crack surfaces (aggregate bridging and interlock), are modeled in the local kth crack coordinate system, named n_k–t_k, whose axes are respectively perpendicular and parallel to crack direction (Figs. 2 and 6e).

As regards aggregate bridging, which generates normal stresses transferred between crack surfaces (Fig. 2a), it is modeled by a smooth curve as a function of the kth crack opening u_k [13,18]:

$$\sigma_{bk} = \frac{f_t}{1 + (u_k/u_0)^p} = c_{bk}\frac{u_k}{a_{mk}} = c_{bk}\varepsilon_k, \tag{1}$$