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Equivalence between stress intensity factor and energy approach based fracture parameters of concrete

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ABSTRACT

The paper presents numerical study and relationship between the double-K fracture parameters and the double-G fracture parameters using two standard tests. The data required for calculation is obtained using cohesive crack model. It is observed that both the corresponding parameters of the double-K fracture model and the double-G fracture model at the onset of crack initiation and unstable fracture are equivalent. This observation agrees well with experimental results available in literature. It is also found that the fracture parameters of double-K fracture criterion and double-G fracture criterion are influenced by initial notch-length/depth ratio, specimen shape, size and softening function.

1. Introduction

The energy dissipation phenomenon due to aggregate bridging stress or cohesive stress distributed across the crack face in fracture process zone (FPZ) has limited the direct application of linear elastic fracture mechanics (LEFM) concept in *quasibrittle* materials like concrete. As a result, several nonlinear fracture models: cohesive crack model (CCM) or fictitious crack model [1–4], crack band model [5], two parameter fracture model [6], size-effect model [7,8], effective crack model [9], double-K fracture model [10–13], the K_R-curve approach based on cohesive stress distribution [10,14] and double-G fracture model [15] have been proposed over the period of time. Cohesive crack model and crack band model are based on finite element or boundary element methods which take into account of fracture process zone by means of constitutive relations showing strain softening and strain localization during the crack propagation. The other six number of fracture models are based on modified form of linear elastic fracture mechanics concept to take the effect of material nonlinear behavior due to existence of the process zone. Fracture models based on modified linear elastic fracture mechanics has a practical advantage that the fracture parameters of concrete can be determined with relatively less computational effort as compared to those based on numerical approach. Crack band model and cohesive crack model are equivalent to each other when the path of the crack (or crack band) is known *a priori* and the band thickness in case of the crack band model is reduced to zero. Cohesive crack model is more popular because of its simplicity and ability to represent the real physical process and describe the nonlinear fracture behavior of the material. The modified linear elastic fracture models are based on stress intensity factor concept except the double-G fracture model which is based on the energy release rate approach. Apart from the external load, crack length and specimen geometry, the fracture energy release rate approach also depends on the deformation characteristic (Young's modulus) of the material. Hence, ductility property is also associated with energy release rate approach unlike that of stress intensity factor based models. Xu and Reinhardt [10] presented three stages of crack propagation in concrete: crack

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Nomenclature

a_o	initial crack length
a_c	effective crack length at peak (critical) load
B	width of specimen
c_1, c_2	material constants for nonlinear softening function
D	depth of specimen
E	modulus of elasticity of concrete
f_t	uniaxial tensile strength of concrete
G_{IC}^{ini}	the crack initiation fracture energy release
G_{IC}^{un}	the unstable fracture energy release
G_{IC}^{C}	the critical value of the cohesive breaking energy
K_C	the critical value of stress intensity factor
K_{IC}^{ini}	the crack initiation toughness
K_{IC}^{un}	the unstable fracture toughness
$\overline{k_{ic}^{un}}$	the effective unstable fracture toughness
$\overline{k_{ic}^{ini}}$	the effective crack initiation toughness
K_{IC}^{C}	the cohesive toughness
S	span of beam
P_{ini}	crack initiation load
P_u	maximum applied load
σ	cohesive stress
υ	the Poisson's ratio
w_g	self-weight of the beam per unit length

initiation, stable crack propagation and unstable crack propagation based on experimental results of large size compact tension (CT) specimens and small size three-point bending test (TPBT) specimens. The different stages of crack propagation can be described by double-K fracture parameters: initial cracking toughness K_{IC}^{ini} and unstable fracture toughness K_{IC}^{un} unlike two parameter fracture model, size-effect model and effective crack model. The value of K_{IC}^{un} is defined as the ability to resist maximum external load at critical fracture condition, whereas the value of K_{IC}^{ini} provides information of external load at which the crack will begin to advance in stable manner. It is revealed from the tests that both the parameters K_{IC}^{ini} and K_{IC}^{un} are dependent on material properties and not on specimen geometry and size. Later, an analytical method was proposed to determine the double-K fracture parameters from three-point bending test, compact tension test and wedge-splitting test specimens of mode I fracture [11,12]. In order to determine the value of cohesive toughness in double-K fracture model, a specialized numerical treatment is needed because of the singularity problem at the integral boundary. A simplified approach [13] was later proposed using two empirical formulae to obtain the double-K fracture parameters for three-point bending configuration which avoided the need of specialized numerical technique for determining the values of cohesive toughness and trial and error approach for computing the value of effective crack length during crack propagation. Further, Kumar and Barai [16,17] proposed a closed form solution of cohesive toughness based on universal form of weight function for determining the double-K fracture parameters using three-point bend, compact tension and wedge-splitting tests geometries. Hence, the weight function approach facilitated to avoid the need of specialized numerical technique without loss of accuracy. Kumar and Barai [18] also presented a comparative study on determination of double-K fracture parameters using analytical method [11,12], simplified approach [13] and the weight function approach for three-point bending and compact tension geometries at different values of initial notch-length/depth (a_o/D) ratio. In the study, the influence of specimen geometry on double-K fracture parameters was numerically carried out for specimen size-range $100 \leqslant D \leqslant 600$ mm and it was reported that these fracture parameters were influenced by size-effect, initial crack length/depth ratio and specimen geometry.

Recently, Xu and Zhang [15] proposed the double-G fracture criterion using the concept of energy release rate which consists of two characteristic fracture parameters: the initiation fracture energy release G_{IC}^{ini} and the unstable fracture energy release G_{IC}^{un}. The value of G_{IC}^{ini} is defined as the Griffith fracture surface energy of concrete mix in which the matrix remains still in elastic state under the initial cracking load P_{ini} and the initial crack length a_o. Until the initial cracking load, the load–displacement (P–δ) or load–crack mouth opening displacement (P–CMOD) curve is linear and the crack driving energy is less than the energy required to create a new crack surface. Once the load value P on the structure is increased beyond to the value of P_{ini}, a new crack surface (macro-cracking) is formed and the cohesive stress along the new crack surface starts acting. The cohesive stress provides additional resistance to the stable crack propagation in terms of cohesive breaking energy G_I^C until the critical condition is achieved. At the onset of unstable crack propagation or critical condition, the total energy release G_{IC}^{un} consist of initiation fracture energy release G_{IC}^{ini} and critical value of the cohesive breaking energy G_{IC}^{C}. Xu and Zhang [15] presented the equivalence relationship between the double-G fracture parameters and the double-K fracture parameters based on extensive test results using two specimen geometries, i.e., three-point bending test of size-range 150–500 mm