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Note

Plastic collapse moment equations of throughwall axially cracked elbows subjected to combined internal pressure and in-plane bending moment

J. Chattopadhyay *, W. Venkatramana, B.K. Dutta, H.S. Kushwaha

Reactor Safety Division, Hall-7, Bhabha Atomic Research Center, Trombay, Mumbai 400 085, India

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A B S T R A C T

Plastic collapse moment (PCM) equations of throughwall axially cracked (TAC) elbow subjected to in-plane closing/opening bending moment were previously proposed by the present authors. However, in actual situation, an elbow may often be subjected to combined internal pressure and bending moment loading. The present work investigates the effect of internal pressure on the in-plane plastic collapse moment of throughwall axially cracked elbows through 3-D elastic–plastic finite element analysis. Equations of un-pressurized cases are recommended where it is conservative and in other cases new equations are proposed.

1. Introduction

Plastic collapse moment (PCM) equations of throughwall axially cracked (TAC) elbow subjected to in-plane closing/opening bending moment were previously proposed by Chattopadhyay et al. [1]. However, in actual situation, an elbow may often be subjected to combined internal pressure and bending moment loading. In continuation of the previous study, the present work, therefore, investigates the effect of internal pressure on the in-plane PCM of TAC elbows through 3-D elastic-plastic finite element analysis. The relevance of a throughwall crack subjected to internal pressure is there in a pressurized heavy water nuclear reactor (PHWR), because leakage through a tight crack is generally small, which do not allow the pressure to fall significantly due to large inventory of water in the piping system. Whatever little fall of pressure may be there, it is compensated by pressurizer and feed–bleed system in a typical PHWR. A total of 147 cases (see Table 1) of long radius ($R_b/R = 3$) elbows with various sizes of axial cracks ($a/D_m = 0$–1.0), different wall thickness ($R/t = 5$–20), and two different bending modes, namely closing and opening have been considered in the analysis, where a is the semi-axial crack length, R_b is the mean bend radius of elbow and R, D_m and t are the mean radius, mean diameter and wall thickness of the elbow cross section respectively. Since crack-closing effect is observed, contact analyses are carried out. Both material and geometric non-linearity is considered to capture the ovalisation of elbow during deformation. Elastic-perfectly plastic stress-strain response of material is assumed. Plastic collapse moments are evaluated from moment – end rotation curves by twice-elastic slope (TES) method. These PCM values are compared with no-pressure cases. For those cases where weakening factors (which is defined as the ratio of PCM of cracked and non-weakened elbow) for pressurized cases are lower than the non-pressurized cases, separate closed-form equations are proposed.

* Corresponding author. Tel.: +91 22 25593775; fax: +91 22 25505151.
E-mail addresses: jchatt@barc.gov.in, jc66in@gmail.com (J. Chattopadhyay).
Abbreviations: PCM, plastic collapse moment; TAC, throughwall axially cracked; TES, twice-elastic slope.

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Nomenclature

a	semi-crack length
D	outer diameter of elbow cross section
D_m	mean diameter of elbow cross section
E	Young's modulus
$h = tR_b/R^2$	elbow factor or pipe bend characteristics
M_0	plastic collapse moment of defect-free elbow
M_L	plastic collapse moment of cracked elbow
P	internal pressure
$p = PR/t\sigma_y$	normalized internal pressure
R	mean radius of elbow cross section
R_b	mean bend radius at elbow crown
t	wall thickness of elbow
$X = M_L/M_0$	normalized plastic collapse moment (PCM) with respect to PCM of defect-free elbow
α	semi-axial crack angle (Fig. 1)
σ_y	yield stress

2. Finite element analysis

The basic approach to investigate the effect of internal pressure on the in-plane PCM of TAC elbows has been same as followed for non-pressurized cases [1]. The finite element method invoking contact analysis is used to investigate the variation of PCM of TAC elbows under in-plane bending moment. Finite element program ANSYS [2] are used for this study. Details of the methodology are available in [1].

Fig. 1 shows the geometry of a TAC elbow. The elbow is connected with straight pipes of length equal to the six times the mean cross sectional radius. It is important to note that this straight pipe length allows free ovalisation of elbow cross sections. The crack is located at the elbow crown. Table 1 shows different combinations of R_b/R, R/t and a/D_m taken in the study. All are long radius ($R_b/R = 3$) elbows, because it has been shown in [1] that bend radius has not much effect on the weakening factors. Because of symmetry, half of the elbow is modeled. Fig. 2 shows one typical finite element mesh used for the analysis, which is the same standardized mesh used earlier [1]. The mesh consists of 1040–1520 twenty-noded solid elements and 6056–8680 nodes depending on the crack size.

The load in the elbows is split in two components: a constant internal pressure and varying in-plane bending moment monotonically increasing in steps. The pressure is applied in initial 100 load steps and subsequently held constant. Crack face pressure is neglected, as it is believed to have negligible effect on the global behavior of moment-rotation data from which plastic collapse moment is evaluated. Internal pressure is normalized as, $p = PR/(t\sigma_y)$, where P is the applied internal pressure and σ_y is the material yield stress. The following material data: yield stress $(\sigma_y) = 300$ MPa, Young's modulus $(E) = 200$ GPa and Poisson's ratio $(\nu) = 0.3$ are used throughout the analysis. However, specific values of these parameters do not affect the results for other cases as all the results are expressed in normalized form. The various normalized pressures considered in the analysis are $p = 0.1$, 0.2 and 0.4. Closed end condition is simulated by applying axial pressure of intensity

Table 1
Various geometric and loading parameters of analyzed elbows.

Bending mode: closing
R: 250 mm
R/t: 5, 10, 15, 20
R_b/R: 3
a/D_m: 0, 0.1, 0.2, 0.4, 0.6, 0.8, 1.0
$p = PR/t\sigma_y$ = 0.1, 0.2, 0.4
No. of cases: 84
Bending mode: opening
R: 250 mm
R/t: 5, 10, 20
R_b/R: 3
a/D_m: 0, 0.1, 0.2, 0.4, 0.6, 0.8, 1.0
$p = PR/t\sigma_y$ = 0.1, 0.2, 0.4
No. of cases: 63
Total no. of cases: 147