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Plastic collapse moment equations of throughwall axially cracked elbows subjected to in-plane bending moment

J. Chattopadhyay *, W. Venkatramana, B.K. Dutta, H.S. Kushwaha

Reactor Safety Division, Hall-7, Bhabha Atomic Research Center, Trombay, Mumbai 400 085, India

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Abstract

A throughwall axial crack may develop in an elbow or pipe bend due to service related degradation mechanism. It is very important to know the plastic collapse moment (PCM) of an elbow in the presence of a throughwall axial crack. The existing PCM equations of throughwall axially cracked (TAC) elbows are based on very few test data points of Griffiths without detailed analyses and also the range of applicability of their proposed equations are limited. Further, they do not differentiate between closing and opening modes of bending although deformation characteristics under these two modes are completely different. Therefore, the present study has been undertaken to investigate through 3D elastic–plastic finite element analysis. A total of 84 elbows with various sizes of axial cracks ($a/D_m = 0$–1), different wall thickness ($R/t = 5$–20), different elbow bend radii ($R_b/R = 2, 3$) and two different bending modes, namely closing and opening have been considered in the analysis. Elastic-perfectly plastic stress–strain response of material has been assumed. Both geometric and material non-linearity are considered in the analysis. Crack closing is observed in most of the cases. To capture the crack closure effect, contact analysis has been performed. Plastic collapse moments have been evaluated from moment–end rotation curves by twice-elastic slope method. From these results, closed-form equations are proposed to evaluate plastic collapse moments of elbows under closing and opening mode of bending moment. The predictions of these proposed equations are compared with the test data available in the literature. Matching between predictions and experimental results is found to be satisfactory.

Keywords: Pipe bend; Elbow; Limit load; Plastic collapse; Throughwall; Axial; Crack

1. Introduction

It is important to ensure the integrity of piping components for safe and reliable operation of the power plants. Pipe bends or elbows are commonly used components in a piping system. They are very flexible

Abbreviations: PCM, plastic collapse moment; TAC, throughwall axially cracked; TES, twice-elastic slope.

* Corresponding author. Tel.: +91 22 25593775; fax: +91 22 25505151.
E-mail address: jchatt@barc.gov.in (J. Chattopadhyay).

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Nomenclature

a	semi-crack length
D	outer diameter of elbow cross-section
D_m	mean diameter of elbow cross-section
E	Young's modulus
$h = tR_b/R^2$	elbow factor or pipe bend characteristics
M_0	plastic collapse moment of defect-free elbow
M_L	plastic collapse moment of cracked elbow
R	mean radius of elbow cross-section
R_b	mean bend radius at elbow crown
t	wall thickness of elbow
$X = M_L/M_0$	weakening factor of throughwall axially cracked elbow plastic collapse moment due to the presence of crack
α	semi-axial crack angle (Fig. 1)
σ_y	yield stress
σ_f	flow stress

compared to the straight pipes. Because of this increased flexibility, they often accommodate large displacements arising from the differential thermal movements. However, care must be taken so that deformations of the bend remain predominantly elastic. Otherwise, the resistance to deformation may decrease rapidly leading to the failure of the system. It is, therefore, important to know its limit load (collectively used to indicate either instability or plastic collapse) for the safe operation of the plant. At the limit load, the deformation of the elbow increases without significant increase in load. Elbows may, sometimes, contain cracks due to manufacturing defect or due to service related degradation mechanism. It is very important to know the effect of cracks on the plastic collapse moments (PCM) of elbows for integrity assessment of the piping system, Two crack configurations, namely, circumferential and axial, are normally encountered in elbows. Miller [1] and Zahoor [2] gave closed-form expression of PCM of elbows with throughwall circumferential and axial cracks based on Griffiths' experimental data. Chattopadhyay et al. [3–5] recently proposed closed-form equations of PCM of throughwall circumferentially cracked elbow. Although it is well known [6–8] that throughwall circumferential crack has larger weakening effect compared to axial cracks, the latter is more likely to be present in actual condition due to service related degradation mechanism. An elbow is prone to develop an axial crack at crown due to being subjected to repetitive loading or at extrados due to erosion. The existing equations of Miller [1] and Zahoor [2] to evaluate PCM of throughwall axially cracked (TAC) elbow do not differentiate between opening and closing mode of bending although the responses of elbows are markedly different under these two different modes of bending moment [3–5,8–10]. Additionally, their equations were developed based on too few test data points of Griffiths [6] without detailed analyses and also the range of applicability of their proposed equations were limited.

2. Scope of the present work

The present study has been undertaken to evaluate the collapse load of TAC elbows subjected to in-plane closing/opening bending moments through 3D elastic–plastic finite element analysis. A total of 84 cases of elbows with various sizes of axial cracks ($a/D_m = 0$–1.0), different wall thickness ($R/t = 5$–20), different elbow bend radii ($R_b/R = 2, 3$) and two different bending modes, namely closing and opening have been considered in the analysis, where a is the semi-axial crack length, R_b is the mean bend radius of elbow and R, D_m and t are the mean radius, mean diameter and wall thickness of the elbow cross-section, respectively. Since crack-closing effect is observed, contact analyses is carried out. Both material and geometric non-linearity is considered to capture the ovalisation of elbow during deformation. Elastic-perfectly plastic stress–strain response of material is assumed. Plastic collapse moments are evaluated from moment–end rotation curves by twice-elastic