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An implicit stress gradient plasticity model for describing mechanical behavior of planar fiber networks on a macroscopic scale

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ABSTRACT

The plasticity behavior of fiber networks is governed by complex mechanisms. This study examines the effect of microstructure on the macroscopic plastic behavior of two-dimensional random fiber networks such as strong-bonded paper. Remote load is a pure macroscopic mode I opening field, applied via a boundary layer assuming small scale yielding on the macroscopic scale. It is shown that using a macroscopic classical homogeneous continuum approach to describe plasticity effects due to (macroscopic) singular-dominated strain fields in planar fiber networks leads to erroneous results. The classical continuum description is too simple to capture the essential mechanical behavior of a network material since a structural effect, that alters the macroscopic stress field, becomes pronounced and introduces long-ranging microstructural effects that have to be accounted for. Because of this, it is necessary to include a nonlocal theory that bridges the gap between microscopic and macroscopic scales to describe the material response in homogeneous continuum models. An implicit stress gradient small deformation plasticity model, which is based on a strong nonlocal continuum formulation, is presented here that has the potential to describe the plasticity behavior of fiber networks on a macroscopic scale. The theory is derived by including nonlocal stress terms in the classical associated J_2-theory of plasticity. The nonlocal stress tensor is found by scaling the local Cauchy stress tensor by the ratio of nonlocal and local von Mises equivalent stresses. The model is relatively easy to implement in ordinary finite element algorithms for small deformation theory. Fairly good agreements are obtained between discrete micromechanical network models and the derived homogeneous nonlocal continuum model.

1. Introduction

It is known that classical homogeneous continuum mechanical descriptions of fiber-based network materials, such as paper, textiles or human tissue, cannot fully describe deformations near macroscopic defects, cf. [1]. The fibers introduce long-ranging microstructural effects that alter the stress field on a macroscopic scale when the inhomogeneous body becomes loaded. The fundamental mechanism behind this phenomenon has received little attention in the literature. There are several candidate mechanisms that may alter the deformation field in a network, as compared to an ideal homogeneous continuum, such as plastic straining of the fibers, fiber and bond breakage or wrinkling and buckling.

In tensile experiments on fiber-based materials such as paper it is well known that the stress–strain curve will deviate from linear behavior and upon unloading there will be a permanent deformation, commonly referred to as plasticity. Hence, in the context of elastic–plastic fiber-based materials, a plastic process is defined as the irreversible process when there is an

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Nomenclature

$x_1, x_2; r, \theta$ Cartesian coordinates; polar coordinates
$\Omega; R$ problem domain; radius to boundary
$\overline{(\cdot)}; \widetilde{(\cdot)}$ nonlocal counterpart of variable $(\cdot)$; dimensionless nonlocal variable
$l; l_0$ characteristic internal length; theoretical estimation of l
$\sigma_0; \sigma_e; \phi; \lambda$ yield stress; von Mises equivalent stress; yield surface; Lagrange multiplier
$\sigma_{ij}; s_{ij}$ Cauchy stress tensor; deviatoric Cauchy stress tensor
$\varepsilon^e_{ij}; \varepsilon^p_{ij}; w; u_i$ elastic strain; plastic strain; strain energy density; displacements
$E; v; \mu; L_{ijkl}$ macroscopic properties: Young's modulus; Poisson's ratio; shear modulus; stiffness tensor
$L; h; E_f; \sigma_{0f}$ fiber properties: length; width; Young's modulus; yield stress
$c; L_s; N$ network properties: coverage; average segment length; total numbers of fibers
$K_I; K_{Ip}; K_I^*$ macroscopic stress intensity factor; K_I at onset of plastic flow; K_I at limit for small scale yielding
$\delta_{ij}; \delta(\cdot)$ Kronecker delta; dirac delta function
$G(\cdot); B_0(\cdot)$ Green's function; modified zero-order Bessel function of second kind
$U; U^{\#}$ total potential energy in homogeneous nonlocal model; network model
$r_p; r_p^{\#}$ estimation of macroscopic plastic zone size in homogeneous nonlocal model; network model
$P; P_c$ macroscopic force; force at which full plastic yielding occur

irreversible straining of the fibers themselves, cf. [2,3]. Obviously, this definition relies on the assumption that the fibers can deform irreversible before they eventually break. In particular, for strong-bonded elastic–plastic fiber-based materials (i.e. in situations when the bonds connecting the fibers are strong compared to the fibers) subjected to traction it is reasonable to assume that there is a significant amount of plastic deformation of the fibers before fracture occurs. This is because the strong-bonded fiber-network structure results in a high degree of axial stress in the fibers. Energy dissipation owing to plasticity processes in the network is in this case assumed significant and the macroscopic behavior of the material is strongly non-linear: a behavior that may be captured by an appropriate plasticity theory on the macroscopic scale. Typical examples of strong-bonded elastic–plastic fiber network materials include printing and packaging papers and some textiles.

The literature shows a variety of approaches for studying this problem. Even though some important results have been reported, cf. [4,5], the knowledge of deformation processes in e.g. paper materials is still in its infancy and needs further exploration. For example; linear fracture mechanics and non-linear fracture mechanics have commonly been used to characterize macroscopic failure phenomena of paper possessing certain defects, cf. [6–8]. Despite these efforts, it is still not established in the materials physics community whether the current non-linear fracture mechanics formulation of heterogeneous fiber-based materials, which essentially is imported from theories developed for homogeneous materials, can be directly applied to describe fracture and deformation behavior. One should have in mind that the mechanical behavior on a microscopic level in network materials such as paper are significant different from those in, say, ductile metals, which are closely connected to dislocation movements in the material. Not only are the basic phenomena entirely different (permanent deformation of fibers vs. dislocation movements in atomic lattices), the mechanisms themselves are acting on totally different scales ($\approx$1 mm vs. $\approx$10 Å).

Addressed in this investigation is a nonlocal plasticity effect on the macroscopic scale that originates from the microstructural properties of the material. Firstly, there is a size-defect dependence in a network material originating from pores and fibers that is not captured by classical homogeneous plasticity continuum theories such as the well-known J_2-flow theory. Secondly, since the network consists of relatively stiff fibers connected to other fibers located at remote distances, the fibers themselves introduce nonlocal structural effects in the material as the overlapping fibers apply long-range actions on each other in a complex manner. Thus, as compared to a homogeneous material that often is approximated as a classical continuum, certain inherent lengths influence the mechanical field in a network material.

In the last years a number of gradient, or nonlocal, plasticity theories have been presented (among others: Fleck and Hutchinson [9,10]; Fleck et al. [11]; Gao et al. [12]; Huang et al. [13]; Mentzel and Steinmann [14]). The various proposed theories are fairly different with respect to the structure of the equations, however, each aims to capture boundary layer phenomena related to phase and grain boundaries, or slips in single crystals, within small deformation formulations. Gudmundson [15] formulated a small deformation isotropic strain gradient plasticity theory, where the plastic flow direction is governed by a certain microstress and not the deviatoric Cauchy stress that is assumed in the classical J_2-flow theory. The microstress is assumed work conjugate to plastic strains.

All the abovementioned studies are considered weakly nonlocal in character because the governing equations in a point include gradients explicitly obtained from state variables in an arbitrary small neighborhood of that point. In the present study, however, a strong nonlocal small deformation theory is used, which is derived by implicitly including gradients on a considerable larger distance than in the weakly nonlocal models. This approach is physically motivated since fibers in e.g. cellulosic networks often have the length of millimeters as compared to microns, or even smaller lengths, in the case of dislocation movements in metals, as discussed above.