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Engineering Fracture Mechanics 75 (2008) 1804–1814
Engineering Fracture Mechanics
www.elsevier.com/locate/engfracmech

An implicit gradient application to fatigue of complex structures ☆

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Received 29 November 2006; received in revised form 19 January 2007; accepted 19 January 2007
Available online 6 February 2007

Abstract

This paper presents a procedure to evaluate the stress gradient effect on the fatigue strength of steel welded joints and notched components. An effective stress is calculated by solving a second-order differential equation over all the component (the implicit gradient approach) independently of its geometric shape. The solution is obtained by assuming the isotropic linear elastic constitutive law for the material and the maximum principal stress as equivalent stress. The fatigue behaviour of geometrically complex steel welded joints is analysed and compared with previous fatigue scatter bands obtained for two-dimensional joints. In complex details, the actual critical point is derived from the analysis and is not assumed a priori. Implicit gradient analysis is also used to investigate high-cycle fatigue behaviour in the case of notches.

In addition, it is shown that critical distance approaches can be obtained from the non-local theory by proper choice of the weight function.

Keywords: Implicit gradient; Welded joints; Fatigue; Equivalent stress; NSIF

1. Introduction

In the presence of high stress concentrations (especially at sharp notches or cracks), accurate predictions of the fatigue behaviour of structures are of great interest in mechanics. Unfortunately, the stress field calculated under the hypothesis of linear elastic material in the vicinity of cracks or sharp notches is a singular function of the distance from the notch tip. Therefore, because of the stress singularity, linear elastic peak stress cannot be used to evaluate the safety factor of structures. In the case of welded structures and other sharply notched components, the elastic peak stress usually provides overestimation of the actual stress effects at the notch by predicting strength values lower than the experimentally measured ones. To overcome this problem (particularly in failure predictions of notches), the idea of volume average has been widely used in the literature, both for fatigue [1] and static failure load predictions [2,3]. For fatigue life estimation, an average local stress

☆ This article appeared in its original form *Fracture of Nano and Engineering Structures: Proceedings of the 16th European Conference of Fracture, Alexandroupolis, Greece, July 3–7, 2006* (Edited by E.E. Gdoutos, 2006). Springer, Dordrecht, The Netherlands. ISBN 1-4020-4971-4.

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0013-7944/$ - see front matter
doi:10.1016/j.engfracmech.2007.01.020

can properly address several types of components and stress raisers (notches, cracks and defects); the average can be obtained along lines or in areas or volumes using slightly different methods usually called "critical distance" approaches [4–6].

For fatigue assessments of welded structures, various methods have been proposed to estimate fatigue life in complex details: the crack propagation approach [7,8], the hot spot method [9], the effective notch stress by imposing a fictitious notch root radius of 1 mm [10], and the Notch Stress Intensity Factors (NSIF) approach [11,12]. In particular, the NSIF approach considers a weld toe or root as an ideal V-notch with linear flank and notch root radius equal to zero, so that Williams' equations and the NSIFs (calculated according to the definition of Gross and Mendelson) can be successfully used for total fatigue life estimation of welded components. The main problem of the NSIF approach is its different physical dimension when the opening angle is changed, so that failure of toes and roots must be dealt with separately; a possible way to overcome this problem is to use the average stress approach or, similarly, to define an average strain energy which can be used as reference parameter (provided that a proper structural volume is defined around the notch tip) for the averaging operation [13,14].

In an apparently different research field, the static behaviour of brittle materials with sharp notches [3,15] was recently investigated using an implicit gradient approach [16,17]. The problem of stress singularity at the tip of sharp V-notches was overcome simply by adopting the linear elastic hypothesis. An interesting peculiarity of the implicit gradient approach is that an effective stress is defined by averaging the stress field and is calculated by means of a partial differential equation in which the second-order spatial derivatives (Laplacian) of the effective stress are used. Thus, to estimate the failure load of components, the effective stress is compared with the ultimate tensile stress of the material, so that failure occurs when the peak of the effective stress reaches the ultimate tensile strength of the assessed material.

In the case of fatigue loading, the implicit gradient approach has already been applied to simple two-dimensional steel welded structures [17]. In [17], a fatigue scatter band was evaluated for cruciform joints independently of the welded angle and type of loading (traction or bending). For more complex welded structures, the implicit gradient approach was used to directly obtain the maximum value of the effective stress range and the location of the point where the fatigue crack initiates.

The aim of the present paper is to extend the implicit gradient approach to fatigue assessments of a wider set of geometries: here we consider rounded notches under fatigue loadings and three-dimensional welded structures. The non-local equivalent stress is calculated by means of an accurate numerical analysis and the whole components are analysed, so that the actual critical points are derived from the numerical investigation and not assumed a priori. In addition, it is shown that the various critical distance approaches (line method, point method, area method and volume method) can be seen as different developments of the same non-local stress framework supporting the implicit gradient approach. In particular, any critical distance approach can be obtained by means of a proper choice of a weight function defined over the whole component.

2. Implicit gradient approach

Let us consider a body of volume V and the linear elastic solution of the stress over V. According to the non-local theory as proposed by Pijaudier-Cabot and Bažant [18,19], an effective stress σ_{eff} in the actual point P can be defined by averaging of its elastic local counterpart, called equivalent stress σ_{eq}, weighted by a function α:

$$\sigma_{\mathrm{eff}}(P) = \frac{\int_V \alpha(P,Q)\sigma_{\mathrm{eq}}(Q)\,\mathrm{d}V}{\int_V \alpha(P,Q)\,\mathrm{d}V} \tag{1}$$

where Q is a point inside volume V and the equivalent stress σ_{eq} is a function of the stress tensor. The weight function α is an isotropic function of the distance $|PQ|$, which vanishes as distance $|PQ|$ increases.

To overcome the evaluation of integral (1) over all volume V for each point $\mathbf{P}$ of V, Peerlings et al. [20] proposed an alternative calculation. After a Taylor expansion of Eq. (1), the evaluation of effective stress σ_{eff}, when α is a proper isotropic function, can be substituted by solution of a differential equation in volume V (for more details, see Refs. [17,20]):