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# Approximation of curved cracks under mixed-mode loading

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**Abstract**

The calculation of stress intensity factors or mechanical energy release rate for non-straight cracks can be complicated. Approximation to equivalent crack shapes can simplify calculations considerably, but this requires an understanding of the influence of key shape parameters on crack-tip stresses. A simple analytical model has been developed, based on the concept of a relaxed volume, to predict mechanical energy release rate and deflection angle for a range of crack shapes under mixed-mode loading. Results from this model compared well with those obtained from finite element (FE) simulations, and with predictions from previous analytical models. It was found that the crack length and orientation of the crack-tip with respect to loading direction are the key influences on fracture parameters, whilst curvature near the crack-tip can also affect results.


*Keywords:* Crack path; Mechanical energy release rate; Crack deflection; Finite element analysis; Mode-mixity

## 1. Introduction

The development of fracture mechanics has been based on straight cracks. Whilst an initial flaw under mode I loading will generally propagate in a straight line, a crack under mixed mode loading will deviate from its original direction, thereby undergoing a change in shape. Understanding of curved cracks under mixed-mode loading is an important issue for structural integrity, particular for structures subjected to complicated or variable loadings [1,2], and for cracks near material interfaces [3]. Prediction of the initial angle of crack deflection under mixed-mode loading has been the focus of much work [4,5]. It is generally understood that expressions developed for stress intensity factors (SIFs) of straight cracks cannot be applied to non-straight cracks. Work on more appropriate expressions, however, has generally been limited to simple crack shapes [5–7]. Alternately, it has required complicated mathematics and computation methods, such as complex stress potentials and conformal mapping [8], weight functions [9] or finite element analysis [2,10,11].

Cotterell and Rice [5] examined the stress field around a slightly kinked crack, branched cracks have been variously analysed [7,12–14] and Sih et al. [6] calculated SIFs for circular-arc cracks. For severely deviating

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cracks with irregular shapes, as observed in highly asymmetric loading situations, solutions for idealised crack-shapes are no more useful than those for straight cracks [2]. Chen and co-workers [15,16] have developed models for calculating SIFs for arbitrary crack shapes, by dividing the crack into a number of straight sections then using complex stress potentials to relate loading tractions to the crack-opening displacements for each section. Application of models of this type to real specimen geometries has been limited.

This work addresses the approximation of curved cracks to equivalent straight cracks, for the purpose of calculating fracture parameters, such as stress intensity factors and mechanical energy release rates. Although a crack shape may be quite complicated, it is believed that the stress intensity factor may be approximated by a function of several simple crack-shape parameters. This has been suggested by several researchers, and shown to be appropriate for particular crack-shapes and loading configurations, although reasons were not given for why this is the case [17–21].

Kitagawa et al. [17] used stress potentials and conformal mapping to investigate various bent (kinked) and branched internal cracks. They suggested the approximate replacement of bent cracks with equivalent straight cracks, and showed that stress intensity factors could be predicted fairly accurately over a wide range of kink lengths and angles. They found discrepancies between the bent crack and equivalent straight crack were highest for small kink lengths, probably due to the interaction between the kink and the main crack. They found that circular arc cracks were not as accurately approximated, which was attributed to curvature at the crack tip.

Noda and co-workers [20,21] modelled various curved edge cracks using a hypersingular integral equation formulation. They showed that a curved edge crack under mode I loading could be simulated by a simplified crack with the same projected length and crack-tip angle, and thereby concluded that curvature at the crack-tip has a negligible effect on the stress intensity factors. They addressed the possibility of the crack deflecting from its original orientation, and found for a circular arc edge crack that the SIF in the direction perpendicular to tensile stress was very close to that of a straight edge crack with the same projected length. The apparently contradictory conclusions regarding the effect of crack-tip curvature may be due to differences in crack configuration (internal versus edge cracks).

The issue of why straight cracks can be used to approximate more complicated shapes was not discussed in any detail by Kitagawa et al. or Noda et al. Rather it was implicitly assumed that if a curved crack may be approximated by a simpler shape, then a straight crack would be the logical shape to use. Approximation to an equivalent straight crack simplifies calculation appreciably and may often be sufficiently accurate. In fact, this approximation is made when treating any macroscopically straight crack as being perfectly straight, despite the fairly ubiquitous presence of crack deflection at a microscopic level. The validity of the approximation in this case is generally not questioned. For more significantly deflecting cracks, this approximation is clearly useful for simply estimating fracture parameters, and thus warrants further investigation.

In this work, an analytical model, based on a relaxed volume concept, was used to predict mechanical energy release rates and crack deflection angles for curved or deviating cracks, from a small number of crack-shape parameters. This was verified with finite element (FE) simulations, and by comparison to results from existing analytical models in the literature. This model attempts to provide an intuitive explanation of why curved cracks may be approximated by equivalent straight cracks.

## 2. Analytical model

### 2.1. Crack shape parameters

It is proposed that the shape of a crack may, for the purposes of calculating a stress intensity factor (SIF), be described by several parameters, from which an approximate SIF value may be predicted. The geometry of a deviating edge-crack is shown in Fig. 1, along with the key crack shape parameters describing crack and loading configuration. The crack axis is defined as parallel to the crack at the edge. For this common configuration in experimental fracture mechanics, the proposed shape parameters are

- crack length, a, parallel to the crack axis;
- transverse deviation, da, perpendicular to the crack axis;
- crack-tip orientation, $\phi$, relative to loading direction;