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A fracture mechanics analysis of cutting and machining

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ABSTRACT

The process of cutting and machining is analysed using concepts developed in the fracture analysis of beam specimens. Increasing cutting forces and decreasing tool rake angles lead to a sequence of deformation processes from elastic bending to elastic–plastic bending and finally to shear yielding in the chip. The conditions for each mode of deformation are identified. Fracture toughness is included in the analysis as is, in addition, the notion of root rotation at the crack tip. Under some circumstances this gives rise to the condition of the tool tip touching the crack tip during which energy is transferred directly to the fracture process. The tool–chip interface is characterised by Coulomb friction and by the inclusion of an adhesion toughness to model the effects of hot polymeric chips sticking to the rake face of the tool. The combined effects of bending and shearing leading to chip curling and coiling are also analysed.

1. Introduction

The analysis of machining processes has a long history (e.g. [1–3]) with several reviews available (e.g. [4]). The motivation was to improve machine tools and the analysis focussed on describing friction effects and the plastic deformation which occurs during chip formation [5–7]. The latter was the subject of considerable effort using slip line theory and attempts were made to resolve the various solutions via experimental studies [8].

The issue of whether fracture and cracks played any part in the process has a complex history. This was proposed by Reuleaux in 1900 [9] but was subsequently abandoned because, erroneously, the energies were said to be too small to warrant inclusion. In addition, the absence of visible cracks was thought to be important. Atkins has firmly established that these omissions were wrong and that the fracture terms are important in any cutting analysis [10,11].

The similarity of cutting a layer from a surface and splitting a cantilever specimen with a wedge has been observed previously [12] and it is therefore possible to take analyses developed for double cantilever beam specimens and apply them to cutting. This includes the notion of root rotation at the crack tip and plasticity in the arms [13]. In this work, these ideas have been explored in greater detail and have been extended to include the case of when the tool tip touches the crack tip. The inclusion of shear yielding in the beam (chip) and the issue of chip curling are also addressed.

Fig. 1 shows the three stages of the deformation as the load is increased and/or the rake angle of the tool is decreased. In Fig. 1a the deformation is elastic, apart from a thin layer of plastic deformation on the inner chip surface and on the cut surface of the workpiece. In such cases the chips are straight, apart from a small amount of inward curvature arising from the plasticity in the chip. In Fig. 1b the loads have increased sufficiently to involve plastic bending in the arm which creates chip

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Nomenclature

Greek alphabets

α	rake angle
α'	change in rake angle
$\overline{\alpha}$	effective rake angle
β	friction angle
χ	$\frac{\Delta}{h} = 0.64$
Δ	correction length
γ_1, γ_2	functions of k_0
γ_3	$2\chi e_Y \tan\alpha$
k_0	$\frac{R_p}{R_0}$
μ	coefficient of friction
ψ	surface slope
v	vertical displacement
σ	stress
σ_Y	yield stress
θ_0	crack tip rotation
$\overline{\theta}$	crack tip rotation for touching condition
$\overline{\theta}_{01}$	$\overline{\theta}_0$ with constant R at R_0 for crack tip touching
$\overline{\theta}_{02}$	$\overline{\theta}_0$ for frictionless conditions for crack tip touching
ω	rotation length

English alphabets

b	cutting width
c	height of elastic zone
e	strain
e_s	shear strain
e_Y	yield strain
$\overline{e}$	max bending strain at the touching condition
E	Young's modulus
E'	plane strain value of E
F_c	force in direction of cut
F_t	force transverse to direction of cut
G	energy release rate
G_a	adhesion toughness
G_b	energy release rate due to bending
$\overline{G}_b$	G_b for crack tip touching
G_e	energy release rate due to touching
G_t	energy release rate due to F_t and root rotation
$\hat{G}$	$\sigma_Y^2 h/2E$
h	cut thickness
$\hat{h}$	critical depth for the touching non-touching transition
H	$\frac{2G_a}{\sigma_Y h(\cos\alpha+\mu\sin\alpha)}$
L	chip length
M	bending moment
M_0	bending moment at point 0
M_p	plastic collapse moment
N	normal force on the tool face
R	radius of curvature
R_0	radius of curvature at point 0
R_p	minimum radius for elastic deformation
S	shear force on the tool face
$\hat{L}$	length of chip where plastic bending ceases
U_{ext}	external work done
U_d	dissipated energy
Z	$\frac{\mu-\tan\alpha}{1+\mu\tan\alpha} = \tan(\beta-\alpha)$

curling, some of which is recovered elastically. The 'crack tip touching' condition is reached when the tool tip reaches the crack tip. This is also shown in Fig. 1b. In Fig. 1c a further increase in load or decrease in rake angle induces a shear