Available online at www.sciencedirect.com
ScienceDirect
Engineering Fracture Mechanics 74 (2007) 2881–2891
Engineering Fracture Mechanics
www.elsevier.com/locate/engfracmech

Note

Elastic T-stress solutions for flat elliptical cracks under tension and bending

V.S. Kirilyuk *, O.I. Levchuk

S.P. Timoshenko Institute of Mechanics, National Academy of Science of Ukraine, Nesterova Str. 3, 03057 Kiev, Ukraine

Received 3 October 2006; received in revised form 18 December 2006; accepted 2 January 2007
Available online 10 January 2007

Abstract

Exact solutions for elastic T-stress of a flat elliptical crack in an infinite body under tension and bending are obtained in this paper. Many papers have been devoted to the problems for elliptical cracks in an elastic medium, but all their attention has been concentrated on the determination of stress intensity factors. In the current paper, elastic T-stress solutions are derived by means of the potential method and a specific collection of harmonic functions. The formulas of the elastic T-stress for a penny-shaped crack [Wang X. Elastic T-stress solutions for penny-shaped cracks under tension and bending. Engng Fract Mech 2004;71:2283–98] follow from the present results as a special case. It is obtained that under tension loading, the elastic T-stress is always compressive along the elliptical crack front. In both tension and bending cases, T-stress essentially depends on the Poisson's ratio of the material, a parametric angle and semi-axes of the ellipse.

Keywords: Elastic T-stress; Constraint effects; Flat elliptical crack; Potential method; Stress intensity factors

1. Introduction

Consider an isotropic linear elastic body which contains a three-dimensional crack under symmetric loading with respect to crack plane. The basic terms in a series expansion of the stress field near the crack front can be described as [1,2]:

$$\sigma_{nn} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\alpha}{2}\left(1-\sin\frac{\alpha}{2}\sin\frac{3\alpha}{2}\right)+T, \quad \sigma_{zz} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\alpha}{2}\left(1+\sin\frac{\alpha}{2}\sin\frac{3\alpha}{2}\right), \tag{1a}$$

$$\sigma_{tt} = \frac{K_I}{\sqrt{2\pi r}}2\nu\cos\frac{\alpha}{2}+E\varepsilon_{tt}+\nu T, \quad \sigma_{nz} = \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\cos\frac{3\alpha}{2}, \quad \sigma_{zt} = \sigma_{nt} = 0, \tag{1b}$$

where (n,t,z) is a local orthogonal coordinate system, which is connected to the crack front point (a local coordinate system formed by the plane normal to the crack front and the plane tangential to the crack front

* Corresponding author. Tel.: +380 44 454 7715; fax: +380 44 456 0319.
E-mail address: kirilyuk_v@ukr.net (V.S. Kirilyuk).

0013-7944/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engfracmech.2007.01.002

Nomenclature

a, b	semi-axes of the elliptical crack
K_{I}	mode I stress intensity factor
v	Poisson's ratio of the material
μ	shear modulus of the material
(n, t, z)	local orthogonal coordinate system
r, α	local polar coordinates
T	magnitude of elastic T-stress
(ξ, η, ζ)	ellipsoidal coordinates
σ_0	remote tension loading
p_1, q_1	remote bending loads
φ	parametric angle
n_x, n_y	components of the normal vector in the crack plane
J_{ij}	special integrals
$K(k), E(k)$	full elliptical integrals of the first and second kind
$t(\varphi)$	magnitude of normalized elastic T-stress

point s); r and α are the local polar coordinates (see Fig. 1); v is the Poisson's ratio and E is the Young's modulus; K_{I} is the mode I stress intensity factor. The symbol T in Eqs. (1a) and (1b) designates the elastic T-stress, corresponding to a tensile/compressive stress acting parallel to the crack plane.

The elastic T-stress is an important parameter, which describes stress states near the crack tip. As it was demonstrated in [3–5], the size and shape of the plastic zone of the plane strain crack-tip and the crack tip constraint depend on sign and magnitude of the T-stress. The T-stress, in combination with the stress intensity factors (SIFs), ensures a two-parameter characterization of linear elastic crack tip fields [6–8]. Such two-parameter approach in linear elastic fracture mechanics is becoming more and more recognized [9,10]. The influence of T-stress on the constraint of interface cracks was investigated in [11].

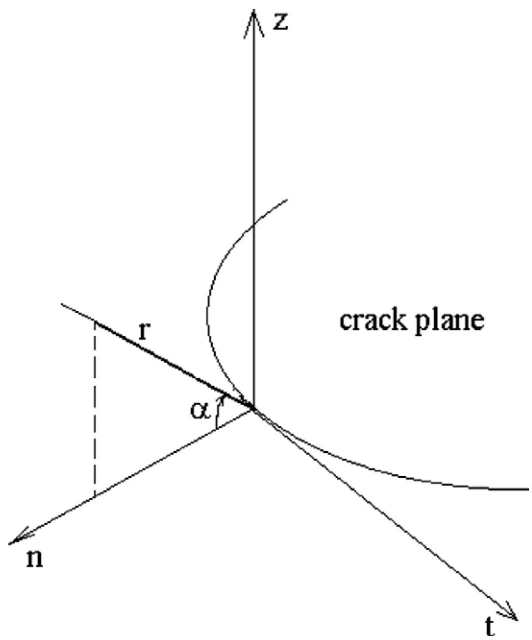

Fig. 1. Local coordinate system along the crack flat.