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Determination of higher order coefficients and zones of dominance using a singular integral equation approach

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Abstract

A method to determine higher order coefficients from the solution of a singular integral equation is presented. The coefficients are defined by $\sigma_{rr}(r,0)=\sum_{n=0}^{\infty}k_n(2r)^{n-\frac{1}{2}}+T_n(2r)^n$, which gives the radial stress at a distance, r, in front of the crack tip. In this asymptotic series the stress intensity factor, k_0, is the first coefficient, and the T-stress, T_0, is the second coefficient. For the example of an edge crack in a half space, converged values of the first 12 mode I coefficients (k_n and T_n, $n=0,\ldots,5$) have been determined, and for an edge crack in a finite width strip, the first six coefficients are presented. Coefficients for an internal crack in a half space are also presented. Results for an edge crack in a finite width strip are used to quantify the size of the k-dominant zone, the kT-dominant zone and the zones associated with three and four terms, taking into account the entire region around the crack tip.

Keywords: Asymptotic analysis; Crack tip opening displacement; T-stress; Singular integral equations; Higher order terms; Edge crack

1. Introduction

Williams [1] was the first to express stresses and strains near the tip of a crack in terms of an asymptotic series for small distances from the crack tip. Using the polar coordinate system in Fig. 1a, the asymptotic form of the stresses and displacements for the symmetric, mode I case of loading can be expressed as

$$\sigma_{ij}(r,\theta)=\sum_{n=0}^{\infty}k_n^{\mathrm{I}}(2r)^{n-\frac{1}{2}}f_{ij}^{\mathrm{I}k}(n,\theta)+\sum_{n=0}^{\infty}T_n^{\mathrm{I}}(2r)^{n}f_{ij}^{In}(n,\theta), \tag{1}$$

$$2\mu u_i(r,\theta)=\sum_{n=0}^{\infty}k_n^{\mathrm{I}}(2r)^{n+\frac{1}{2}}g_i^{In}(n,\theta)+\sum_{n=0}^{\infty}T_n^{\mathrm{I}}(2r)^{n+1}g_i^{In}(n,\theta),\quad i=r,\theta, \tag{2}$$

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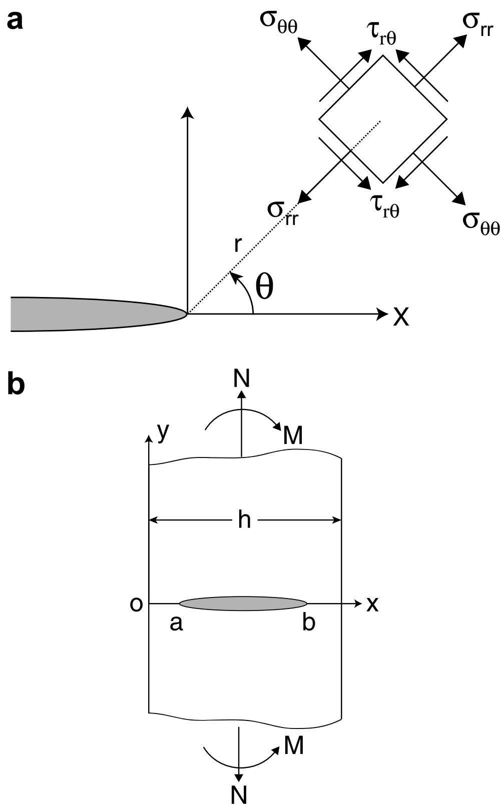

Fig. 1. Problem geometry for the region around the crack tip (a) and for a crack in an infinite strip (b).

where the angular functions for both modes I and II are presented in Appendix A. Within the main body of the paper the superscript “I” will be omitted since only mode I is considered in this study, i.e., $k_n = k_n^{\mathrm{I}}$ and $T_n = T_n^{\mathrm{I}}$. These coefficients are referred to as the stress intensity factor coefficients and T-stress coefficients, respectively.

The application of linear elastic fracture mechanics (LEFM) involves two length scales, one physical and the other mathematical. In terms of a radius of a circle centered at the crack tip, the physical length scale (r_{p}) defines the zone in which all phenomena not accounted for by LEFM occur, while the mathematical length scale (r_{m}) defines the zone in which the truncation of the infinite series, (1) and (2), to one term is adequate. By a Saint Venant’s type of argument, one requires, approximately, $r_{\mathrm{m}} \geqslant 3r_{\mathrm{p}}$ for some acceptable level of error that is used to define r_{m} (see [2]). The focus of the current study is the effect of the higher order terms on the mathematical length scale.

Perhaps the most important work with higher order terms has been with the elastic T-stress, which corresponds to T_0 in (1) and (2). Larsson and Carlsson [3] were the first to demonstrate the importance of this quantity in fracture. They showed that the T-stress has an affect on the plastic zone size and shape, which led these authors to conclude that the T-stress might play a role in characterizing the necessary conditions for fracture in the presence of significant yielding. The work of Levers and Randon [4] provides further evidence of the importance of the T-stress as a secondary fracture parameter, which can be used to explain differences in the fracture behavior of two specimens that are subjected to the same applied stress intensity level. This behavior has been verified in an elastic–plastic analysis by Betegon and Hancock [5], who showed that a negative T-stress tends to lower stresses near a crack tip, which makes the material appear tougher at the same applied J-level than if a positive T-stress exists. This later study follows the important study of Li and Wang [6] who revealed a case in nonlinear fracture where a two-parameter criterion is important; see also the study by Chao and Zhu [7] and references. Loghin and Joseph [8,9] have also shown that in the mixed mode nonlinear