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# Nonlocal anisotropic damage model and related computational aspects for quasi-brittle materials

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**Abstract**

A three-dimensional damage model with induced damage anisotropy is proposed for quasi-brittle materials such as concrete. The thermodynamics framework is used, considering then a single second-order tensorial damage variable whatever the intensity and the sign of the loading. The quasi-unilateral conditions of micro-cracks closure are written on the hydrostatic stress only. Altogether with the consideration of damage laws ensuring a damage rate proportional to the positive part of the strain tensor this is sufficient to model a strongly different behavior due to damage in tension and in compression. A proof of the positivity of the intrinsic dissipation due to such an induced anisotropic damage is given.

An efficient scheme for the implementation of the damage model in commercial Finite Element codes is then detailed and numerical examples of structural failures are given. Plain concrete, reinforced and pre-stressed concrete structures are computed up to high damage level inducing yielding of the reinforcement steels. Local and nonlocal computations are performed.

A procedure for the control of rupture is proposed. It is a key point making the computations with anisotropic damage truly efficient.


*Keywords:* Concrete; Damage; Induced anisotropy; Nonlocal; Finite Element computations

## 1. Introduction

To extend existing isotropic damage models to induced anisotropy is not an easy task as difficulties and questions specifically related to anisotropy arise. How to write the coupling damage/elasticity? What becomes then the effective stress concept associated with the principle of strain equivalence [1,2]? Which tensorial representation of damage shall be used? If general damage anisotropy can be represented by a fourth-order damage tensor [3–5], this formally simple choice is difficult to work with, usually because of the high number of

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material parameters introduced. And how to model properly and efficiently (when numerical computations are in mind) the stiffness recovery due to the micro-defects closure effect [6–11]? the damage growth higher in tension than in compression? In order to give a practical answers to these questions, the choice to represent the damage state by a second-order damage tensor has been made by many authors [12,2,13–16].

Concerning physical damage anisotropy, quasi-brittle materials such as concrete exhibit a micro-cracking pattern different in tensile and in compressive loadings [17,18]: the micro-cracks are mainly orthogonal to the loading direction in tension and parallel to it in compression. This induced anisotropy is responsible for the large dissymmetry tension/compression of concrete behavior and must be introduced to do so in constitutive modeling. For thermodynamics consistency and according to this last remark only one damage variable must be considered. As a state variable a damage variable represents the micro-cracks pattern, whatever the sign of the loading [19,20], and cannot be related to either tension or compression. For the sake of relative simplicity anisotropic damage is next represented by the second-order tensor $\boldsymbol{D}$ of components $D_{ij}$. If a tensile loading is applied in direction 1, induced anisotropic (diagonal) damage shall act as its component $D_1 = D_{11}$ instead of a damage variable "for tension". If the loading is compressive, damage shall act as its component $D_2 = D_{22} = D_{33}$ instead of a damage variable "for compression". Written in the thermodynamics framework the model presented next follows these guidelines.

## 2. Coupling damage/elasticity using a second-order damage tensor

The thermodynamics framework proposed by Ladevèze leading to 3D continuous stress–strain responses is used [6,21,22]. The damage state is represented by the second-order tensor $\boldsymbol{D}$ and there is one known thermodynamics potential $\rho\psi_0^\star$ from which derives a symmetric effective stress $\tilde{\boldsymbol{\sigma}}$ independent from the elasticity parameters [23]:

$$\rho\psi_0^\star = \frac{1+\nu}{2E}\operatorname{tr}(\boldsymbol{H}\boldsymbol{\sigma}^D\boldsymbol{H}\boldsymbol{\sigma}^D) + \frac{1-2\nu}{6E}\frac{(\operatorname{tr}\boldsymbol{\sigma})^2}{1-\eta D_{\mathrm{H}}} \tag{1}$$

with $E$, $\nu$ the Young modulus and Poisson ratio of initially isotropic elasticity, $\eta$ the hydrostatic sensitivity parameter ($\eta \approx 3$ for most materials [22]), $\rho$ the density, where $\boldsymbol{\sigma}^D = \boldsymbol{\sigma} - \frac{1}{3}\operatorname{tr}\boldsymbol{\sigma}\,\mathbf{1}$ is the deviatoric stress and where $\boldsymbol{H}$ is the effective damage tensor, $D_{\mathrm{H}}$ the hydrostatic damage,

$$\boldsymbol{H} = (\mathbf{1} - \boldsymbol{D})^{-1/2} \quad \text{and} \quad D_H = \frac{1}{3}\operatorname{tr}\boldsymbol{D} \tag{2}$$

Quasi-brittle materials such as concrete exhibit a strong difference of behavior in tension and in compression due to damage. This micro-defects closure effect usually leads to complex models when damage anisotropy is considered [24,9,25,26] and the purpose here is to show that for monotonic applications it is sufficient to consider damage anisotropy with a quasi-unilateral effect acting on the hydrostatic stress only, with a thermodynamics potential rewritten

$$\rho\psi^\star = \frac{1+\nu}{2E}\operatorname{tr}(\boldsymbol{H}\boldsymbol{\sigma}^D\boldsymbol{H}\boldsymbol{\sigma}^D) + \frac{1-2\nu}{6E}\left[\frac{\langle\operatorname{tr}\boldsymbol{\sigma}\rangle^2}{1-\operatorname{tr}\boldsymbol{D}} + \langle-\operatorname{tr}\boldsymbol{\sigma}\rangle^2\right] \tag{3}$$

so that the elasticity law reads

$$\boldsymbol{\epsilon} = \rho\frac{\partial\psi^\star}{\partial\boldsymbol{\sigma}} = \frac{1+\nu}{E}\tilde{\boldsymbol{\sigma}} - \frac{\nu}{E}\operatorname{tr}\tilde{\boldsymbol{\sigma}}\,\mathbf{1} \tag{4}$$

and defines the symmetric effective stress $\tilde{\boldsymbol{\sigma}}$ independent from the elasticity parameters,

$$\tilde{\boldsymbol{\sigma}} = (\boldsymbol{H}\boldsymbol{\sigma}^D\boldsymbol{H})^D + \frac{1}{3}\left[\frac{\langle\operatorname{tr}\boldsymbol{\sigma}\rangle}{1-\operatorname{tr}\boldsymbol{D}} - \langle-\operatorname{tr}\boldsymbol{\sigma}\rangle\right]\mathbf{1} \tag{5}$$

with $(\cdot)^D = (\cdot) - \frac{1}{3}\operatorname{tr}(\cdot)\,\mathbf{1}$ the deviatoric part. The notation $\langle\cdot\rangle$ stands for the positive part of a scalar, $\langle x\rangle = \max(x, 0)$.