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Stress intensity factors of two diametrically opposed edge cracks in a thick-walled functionally graded material cylinder

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Abstract

A method is developed to evaluate stress intensity factors for two diametrically-opposed edge cracks emanating from the inner surface of a thick-walled functionally graded material (FGM) cylinder. The crack and the cylinder inner surfaces are subjected to an internal pressure. The thermal eigenstrain induced in the cylinder material due to nonuniform coefficient of thermal expansion after cooling from the sintering temperature is taken into account. First, the FGM cylinder is homogenized by simulating its nonhomogeneous material properties by an equivalent eigenstrain, whereby the problem is reduced to the solution of a cracked homogenized cylinder with an induced thermal and an equivalent eigenstrains and under an internal pressure. Then, representing the cracks by a continuous distribution of edge dislocations and using their complex potential functions, generalized formulations are developed to calculate stress intensity factors for the cracks in the homogenized cylinder. The stress intensity factors calculated for the cracks in homogenized cylinder represents the stress intensity factors for the same cracks in the FGM cylinder. The application of the formulations are demonstrated for a thick-walled TiC/Al_2O_3 FGM cylinder and some numerical results of stress intensity factors are presented for different profiles of material distribution in the FGM cylinder.

Keywords: Functionally graded material; Edge crack; Eigenstrain; Edge dislocation; Thick-walled cylinder

1. Introduction

Functionally graded materials (FGMs) consist of two or more distinct material phases, such as different ceramics or ceramics and metals. The distribution of each material changes continuously with space variables, which introduces nonhomogeneity in the mechanical and thermal properties of these materials. The degree of nonhomogeneity in the mechanical and thermal properties can be controlled by controlling the material distribution of these materials. Because of their outstanding advantages over conventional composites and monolithic materials, these materials have received wide attention of engineers and researchers from different fields of interest. So far, the effects of material distribution on the characteristics of these materials under various

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Nomenclature

A, B constituents of functionally graded material
b_1, b_2 components of Burgers vector
$b_1(s)$, $B(T)$ dislocation density functions
$e^i_{j,e}$ $(j = r, \theta, z)$ elastic strain developed due to equivalent eigenstrain
E, E_0 Young's moduli of FGM and homogenized cylinders, respectively
E_A, E_B Young's moduli of constituent materials
F_I normalized stress intensity factor
h distance between origin and any point of the cylinder
H_i, T_i collocation and integration points, respectively
K_I mode I stress intensity factor
K, K_A, K_B bulk moduli of FGM and constituent materials, respectively
l crack length
n number of layers
p internal applied pressure
P^f_{i-1}, P^f_i resultant pressures at the inner and outer surfaces of the ith layer of FGM cylinder
P^h_{i-1}, P^h_i resultant pressures at the inner and outer surfaces of the ith layer of homogenized cylinder
r radial distance from origin to any point of the cylinder
r_i, r_{i-1} outer and inner radii of the ith layer of the cylinder
R_i, R_o inner and outer radii of the cylinder
s distance of a discrete edge dislocation from inner surface of the cylinder
S_f strength factor
$u^f_i, u^{h_0}_i$ displacements in FGM and homogenized cylinders, respectively
V_A, V_B volume fractions of constituents
z distance of a point (complex variable)
CTE coefficient of thermal expansion
FGM functionally graded material
ε^* thermal eigenstrain
ε^e_j $(j = r, \theta, z)$ equivalent eigenstrain
$\varepsilon^i_{j,e}$ $(j = r, \theta, z)$ equivalent eigenstrain in the ith layer
$\varepsilon^i_{j,f}$ $(j = r, \theta, z)$ strain in the ith layer of FGM cylinder
ε^i_{j,h_0} $(j = r, \theta, z)$ strain in the ith layer of homogenized cylinder
$\sigma^i_{j,f}$ $(j = r, \theta, z)$ stress components in the ith layer of FGM cylinder
σ^i_{j,h_0} $(j = r, \theta, z)$ stress components in the ith layer of homogenized cylinder
$\sigma^i_{j,e}$ $(j = r, \theta, z)$ stresses developed due to equivalent eigenstrain
μ, μ_0 shear moduli of FGM and homogenized cylinders, respectively
μ_A, μ_B shear moduli of constituents
ν, ν_0 Poisson's ratio of FGM and homogenized cylinders, respectively
α, α_A, α_B coefficients of thermal expansion of FGM and its constituents
β angle of dislocation with x-axis
$\Phi(z)$, $\Psi(z)$ complex potential functions for edge dislocation
ΔT change in temperature

loading conditions and for various geometries have been investigated from different points of view. Ootao et al. [1] considered an FGM hollow sphere and optimized the material distribution for the relaxation of thermal stresses. They [2] also considered an FGM hollow cylinder and optimized the material distribution for thermal stress relaxation. Analyses of crack problems of these materials are also carried out with a view to