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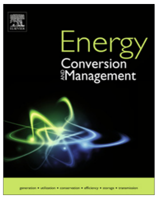

Optimal conductive constructal configurations with “parallel design”

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ABSTRACT

Today, conductive volume to point cooling of heat generating bodies is under investigation as an alternative method for thermal management of electronic chipsets with high power density. In this paper, a new simple geometry called “parallel design” is proposed for effective conductive cooling of rectangular heat generating bodies. This configuration tries to minimize the thermal resistance associated with the temperature drop inside the heat generating volume. The geometric features of the design are all optimized analytically and expressed with simple explicit equations. It is proved that optimal number of parallel links is equal to the thermal conductivity ratio multiplied by the porosity (or the volume ratio). With the universal aspect ratio of $H/L = 2$, total thermal resistance of the present parallel design is lower than the recently proposed networks of various shapes that are optimized with help of numerical simulations; especially when more conducting material is available.

1. Historical background

Today, the increasing integration and power density in electronic devices, has forced the engineers to search for appropriate and effective cooling technologies. The idea of inserting highly conductive links in a heat generating volume was first proposed by Bejan [1] and triggered a large number of studies in this field. The objective is to minimize the temperature of the hot spots in a heat generating volume by decreasing the thermal resistance between that point and a small heat sink placed on the boundary. In order to use the minimum amount of expensive highly conductive material (which itself occupies valuable space), the geometry (configuration) of the conducting tree should be devised carefully.

The pioneer work of Bejan [1] which introduced *the constructal theory* to the literature, was based on geometric optimization of a rectangular heat generating area with a simple I shaped insert (called elemental volume). Then he proposed networks of higher complexity by joining the smaller volumes to a new high-conductivity link, making higher order *constructal trees*.

As the analytical solution of Bejan [1] was derived using some assumptions and simplifications, researchers tried to modify the solution or relax some of the assumptions in conductive cooling of rectangular heat generating bodies [2–13]. Ghodoosi and Egrican [2] dismissed the application of an effective thermal conductivity in Bejan’s [1] solution and solved the temperature drop exactly with one dimensional assumptions. They showed that going to 2nd and higher order constructs is not beneficial. The same results were obtained by Wu et al. [3] by introducing a correct effective thermal conductivity. Moreover, two-dimensional temperature distribution in a rectangular elemental volume was obtained by Kuddusi and Denton [4] analytically. Also, a lower limit for the thermal resistance of the assemblies was found by Wu et al. by employing an infinite number of elemental volumes [5]. Besides, trees with links of variable thickness have also been studied analytically by Zhou et al. [6] and Wei et al. [7].

At the same time, researchers applied computational methods to investigate different configurations of conductive links. The numerical solutions help the engineers relax some simplifying assumptions and add more degrees of freedom to the trees. In case of rectangular bodies, Ledezma et al. [14] simulated constructal networks with different intersecting angles. Almogbel and Bejan [15] investigated trees with spacings at the tip and non-uniformly distributed trees with elemental volumes of different aspect ratios were also optimized by Almogbel and Bejan [16]. Later, Marck et al. [17] used a numerical code to model and evaluate conventional constructal trees of different orders in a fixed rectangular heat generating area.

2. Are the computers the ultimate solution?

In recent years, many studies have focused on trees with various new geometries such as X shaped [18,19],fork shaped [20], Phi and Psi shaped [21], Y shaped [22], V shaped [23,24], “+” shaped [25], three dimensional trees of T and Y shaped branches [26] and asymmetric trees in a rectangle with non-uniform heat

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generation [27] for cooling heat generating *rectangles*. These numerical investigations rely heavily on powerful performance of computers and by nature, do not reveal the physical aspects and features of each design as much. The diversity of the proposed designs also shows that these configurations are usually *assumed* (maybe based on the engineering insight of the designer); and their performance is determined by results of the numerical simulation. In this trend, a limited number of degrees of freedom are *chosen* and by changing one parameter at each level, the multi-times optimal design is obtained after performing a huge number of numerical simulations. Although these studies have led to valuable cooling networks with good thermal performance, but their results cannot be generalized and many questions still left unanswered. For example, most of the above mentioned studies [18–24] have considered a heat generating *square* in their design when the heat sink is on the boundary. However, is a square shaped area the best layout to be cooled in this case? Or aspect ratios other than one might be able to perform better! How the answer to this question is related to the important parameters affecting the solution such as thermal conductivities and amount of the conductive material inserted? Is the optimum geometrical features obtained for a square, still applicable to rectangles of other aspect ratios? Or the whole number of simulations should be repeated for a new layout?

One answer to the above issue might be that as the computers are getting more and more powerful each day; the engineers are less and less concerned about the computational cost of the solutions. This is actually the trend observed in literature today, but it seems that by missing the physical aspects and the important dimensionless groups, the chance of reaching to the global optimum without exhaustive search becomes weaker. Just imagine a world of fluid mechanics equipped with powerful computational tools, with no Reynolds number available!

On the other hand, constructal theory was initially developed with the idea of distribution of internal imperfections optimally to gain the best global performance. To do that, one needs to have simple and general formulations for the imperfections (resistances). The more general, the higher the chance of finding the global optimum is. The simpler, the easier the phenomena can be expressed and analyzed analytically.

The analytical approach of Eslami and Jafarpur [28,29] was developed as a tool to achieve the above goal. In an arbitrary network of highly conductive links, thermal resistance of each link is determined as a function of the geometry of heat generating area and a summation rule gives the total thermal resistance. With this general formulation, the optimal allocation of high conductivity material to each link can be determined explicitly [29] without the need to compare a huge number of numerical simulations.

This method can be used as a guide line toward better performing configurations. As an example, Eslami and Jafarpur [30] showed that how V shaped and pencil shaped designs can be *deduced* as a better alternative to simple I shaped links. (This finding might be the cause of recent attitude toward V shaped and similar designs among researchers [23,24].) It was also proved mathematically that an optimal V shaped network is superior to a corresponding Y shaped tree [30]; a fact that was later approved after a large number numerical simulations in ref [22].

In this paper, a new and simple cooling configuration called the "parallel design" is introduced. This geometry is developed based on the lessons learned from thermal performance of a pencil shaped design [30]. With the theoretical formulation of imperfections in hand [28,29], the configuration is designed in order to minimize the thermal resistance R_g as it will be discussed in the next section. Every geometric feature of the network is determined and optimized analytically. The result is a simple geometry composed of rectangular heat generating areas with parallel high conductivity inserts. This simplicity is extremely important in industrial fabrication of conductive networks at small scale. At the same time, cooling performance of this new design is outstanding. Comparing thermal resistance of the "parallel design" with the already numerically multi-time optimized networks of various geometries in literature shows that simple but optimal distribution of imperfections can lead to systems with superior performance.

3. The parallel design

Fig. 1a shows a low conductivity rectangular body of dimensions $L \times H \times w$ with constant volumetric heat generation. An I shaped link of highly conductive material collects and conducts the heat flow to a point heat sink placed on the boundary. The total thermal resistance in this cooling configuration can be modeled as the summation of two resistances R_g and R_0 as described by Eslami and Jafarpur [28]:

$$R_g = \frac{H/2}{k_0 \times \frac{A_0 w}{H}} \tag{1}$$

$$R_0 = \frac{L_{eff}}{k_p \times (Dw)} \tag{2}$$

where L_{eff} is the effective length of a conductive link which depends on the geometry of the heat generating body and is equal to $L/2$ in this case [28]. As R_g is proportional to square of distance between the point with $T_{\max}$ and the nearest link, Eslami and Jafarpur [30] reasoned that an effective way to reduce the thermal resistance R_g to one fourth is to use the pencil shaped design shown in Fig. 1b. While the thermal resistance associated to the total highly conductive links does not change noticeably, one may think that the resistance R_g can be further reduced by increasing the number

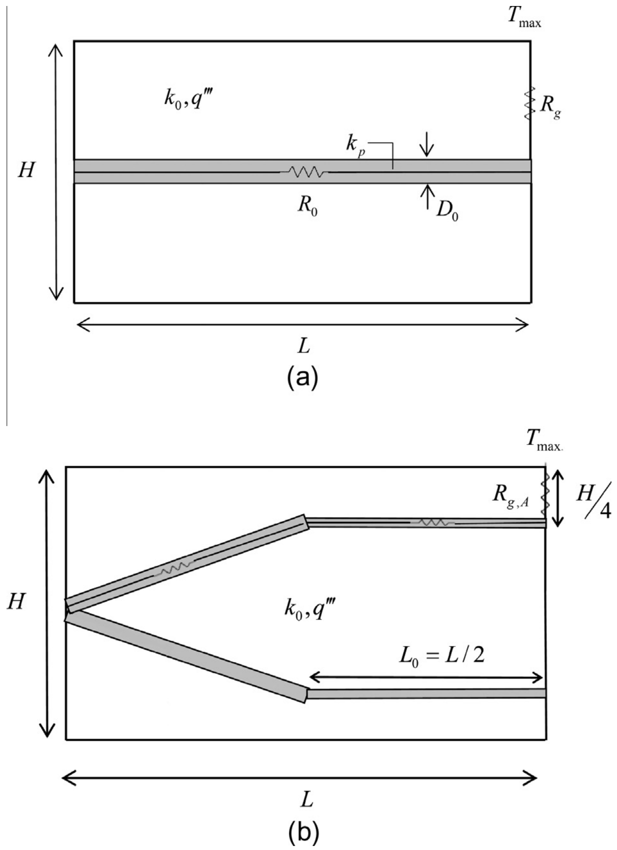

Fig. 1. (a) The I shaped elemental volume. (b) The pencil shaped design [30].