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Modal analysis of multi-cracked beams with circular cross section

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Abstract

This paper proposes a numerical model that combines the finite element and component mode synthesis methods for the modal analysis of beams with circular cross section and containing multiple non-propagating open cracks. The model virtually divides a beam into a number of parts from the crack sections and couples them by flexibility matrices considering the interaction forces that are derived from the fracture mechanics theory. The main feature of the presented approach is that the natural frequencies and mode shapes of a beam with an arbitrary number of cracks and any kind of two end conditions can be conveniently determined with a reasonable computational time. Three numerical examples are given to investigate the effects of location and depth of cracks on the natural frequencies and mode shapes of the beams. Moreover, it is shown through these examples that the evaluation of modal data obtained by the proposed model gives valuable information about the location and size of defects in the beams.

Keywords: Vibration; Multicracks; Finite element analysis; Vibration based detection of cracks

1. Introduction

Mechanical vibrations, long-term service or applied cyclic loads may result in the initiation of structural defects such as cracks in the structures. Accordingly, the determination of the effects of these deficiencies on the vibration safety and stability of the structures is an important task of engineers. Cracks in a structural element modify its stiffness and damping properties. In view of that, the modal data of the structure hold information relating to the place and dimension of the defect. Vibration analysis allowing online inspection is a novel and attractive method to detect cracks in the structures. The effects of the cracks on the dynamical behaviour of the structures have been the subject of many researchers in the past (Cawley and Adams [1], Gounaris and Dimarogonas [2], Shen and Chu [3], Krawczuk and Ostachowicz [4], Ruotolo et al. [5], Kisa et al. [6], Shifrin and Ruotolo [7], Kisa and Brandon [8,9], Viola et al. [10], Krawczuk [11], Patil and Maiti

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Nomenclature

a	crack depth
A	crack area
u, v	displacements with respect to x and y axes
θ	rotation about z axis
m	number of components
L	length of the beam
D	diameter of the beam
R	radius of the beam
R/L	slenderness ratio
a/D	crack depth ratio
L_i/L	crack position
E	Young's modulus of elasticity
v	Poisson's ratio
ρ	material density
K_{I}	stress intensity factor for mode I
K_{II}	stress intensity factor for mode II
K_{III}	stress intensity factor for mode III
J	strain energy release rate
U	strain energy
c_{ij}	flexibility coefficients
C	flexibility matrix
K_i	stiffness matrix for element i
M_i	mass matrix for element i
K_{cr}	stiffness matrix induced by the crack
K_{C}	connection matrix
F	exerted force of a spring
P_i	applied loads
u_i	additional displacement caused by the crack
$\mathbf{q}$	generalised displacement vector
$\mathbf{f}(t)$	external force vector
$\mathbf{g}(t)$	applied force vector
$\mathbf{d}(t)$	spring force vector
$\mathbf{s}$	principal coordinate vector
ω	natural frequency
ω_{cr}	natural frequency of the cracked beam
ω_{nc}	natural frequency of the non-cracked beam
ϕ_{ij}	modal matrix
ψ_{ij}	mass normalised modal matrix

[12], Kisa [13]). Gudmundson [14] investigated the influence of small cracks on the natural frequencies of slender structures by perturbation method as well as by transfer matrix approach. Yuen [15] proposed a methodical finite element procedure to establish the relationship between damage location, damage size and the corresponding modification in the eigenparameters of a cantilever beam. Qian et al. [16] developed a finite element model of an edge-cracked beam. Rizos et al. [17] denoted the crack as a massless rotational spring, whose stiffness was computed by making use of the fracture mechanics. Shen and Taylor [18] presented a detection process to reveal the crack characteristics from vibration measurements.

Most studies on the vibration analysis of circular beams and shafts deal with single crack. Dimarogonas and Papadopoulos [19], by using the theory of cracked shafts with dissimilar moments of inertia, investigated