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A general unified treatment of lamellar inhomogeneities

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Abstract

Consider a lamellar inhomogeneity embedded in an unbounded isotropic elastic medium. When the elastic moduli of the lamellar inhomogeneity are zero it is a crack, if its elastic moduli are infinite it is an anticrack, and when its elastic moduli are finite it is called a quasicrack. Based on the Eshelby's equivalent inclusion method (EIM), the present paper develops a unified approach for determination of the exact closed-form expressions for modes I, II, and III stress intensity factors (SIFs) at the tips of lamellar inhomogeneities under a remote applied polynomial loading.
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Keywords: Lamellar inhomogeneity; Anticrack; Quasicrack; Equivalent inclusion method; Polynomial loadings

1. Introduction

In the context of the present study, a lamellar inhomogeneity is deduced from an ellipsoidal inhomogeneity by letting one of its principal axes vanish. The aim of this paper is to determine modes I, II, and III stress intensity factors (SIFs) pertinent to a general class of lamellar shapes embedded in an infinite isotropic elastic medium under polynomial far-field applied loading in a unified manner.

For a through finding on the SIFs of cracks in the literature, up to the year 2000, one should refer to the handbooks of Murakami [1] and Tada et al. [2]. A close scrutiny of the literature reveals that, except for a very few specific cases, the exact solution to the modes I, II, and III SIFs of three-dimensional penny shape and elliptic cracks under polynomial loading at infinity has not been obtained. Most of the closed-form solutions to the SIF pertinent to a penny shape crack in an infinite isotropic elastic body are devoted to at most linear far-field loading, [2–4]. The more general geometry of an elliptic crack under a uniform far-field tension was considered by Irwin [5] employing the stress function theory. Shibuya [6] also used stress function to study the elliptic crack under a linear far-field loading and evaluated only the mode I SIF within 1% accuracy [1]. As far

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as slit-like cracks are concerned Isida [7] has obtained the closed-form expression only for the mode I SIF at the tip of a two-dimensional crack under a polynomial loading at infinity by stress function method [1].

It is now more than three decades that scientists are concerned with the fiber–matrix force transfer in fiber reinforced composite materials. A well taken approach is to model the long thin fibers by line (lamellar) inhomogeneities [8–14], such that the fiber cross sectional area vanishes. In applying the proposed approach to real composites, if the microgeometries of the reinforcements are such that they can be approximated as limiting cases of an ellipsoid, reasonable estimation by the present method is expected. For example, often a short fiber can be represented by a prolate ellipsoid having the same aspect ratio [15]. Similarly, a long fiber can be modeled by an elliptic cylinder.

Rigid inhomogeneities have important applications in materials science. Nowadays, the excellent technological applications of carbon nanofiber reinforced polymer composites have attracted the attentions of industry and numerous scientists. These composites are advantageous for their high tensile modulus, strength, and promising electrical and thermal properties. Vapor grown carbon nanofiber (VGCF) which is a new class of carbon fiber can be fabricated at high quantities and low cost. The fiber may have a diameter of about 150 nm and length of 10–20 μm [16]. The modulus of carbon nanofiber is normally in the range of 100–600 GPa and sometimes even higher, whereas the modulus of some polymers is usually 2–5 GPa [17]. Thus, for certain purposes the carbon nanofibers in the carbon nanofiber reinforced polymer composites may be considered as rigid fibers or anticracks in the context of the present study. A brief examination of typical properties of fibers and matrices which are now extensively used in composite materials reveals that, many types of fibers such as SiC, Al_2O_3, high modulus (HM) carbon, etc. (in the forms of platelets, whiskers, powder, etc.) have high stiffness in comparison to thermosets and thermoplastics matrices such as epoxy resins, polyesters, and polypropylene. Elastic deformation of these composites with different fiber architecture (long-fiber, short-fiber, ribbon-like fiber, etc.) can be drawn assuming the linear-elastic stress–strain behavior [15]. Moreover, non-metallic and very hard embrittlements in some metallic materials and alloys can be idealized as rigid imperfections relative to the matrix and linear-elastic analysis has proved to be useful in such situations, Hasebe et al. [18,19]. Same authors have considered an anticrack as a model for a thin rigid plate embedded in an elastic body. Atkinson [8], in an effort to determine the elastic fields of a metallic strain measuring device embedded in a rubber matrix, proposed ribbon-like inhomogeneity model. In this, Atkinson examines both cases of a rigid ribbon inhomogeneity (anticrack) and an elastic ribbon inhomogeneity (quasicrack). A considerable amount of literature is devoted to the two-dimensional problem of anticrack embedded in an isotropic elastic materials under uniform far-field loading, Hasebe et al. [18,19], Dundurs and Markenscoff [20], Markenscoff and Ni [21], Hurtado et al. [22], and Homentcovschi and Dascalu [23]. However, to the best of the authors' knowledge, the three-dimensional cases of penny shape and elliptic anticracks under far-field polynomial loading have not been addressed in the literature. Also, to date, very little attention has been paid to the determination of the stress field of quasicracks. The existing theories devoted to this topic are tailored for the simpler two-dimensional and uniform loading conditions and have very limited ranges of applicability. For example, Hurtado et al. [22] who address quasicracks, consider the in-plane and out-of-plane cases under a uniform far-field loading.

Depending on the dimension of the problem, loading condition, and the type of lamellar inhomogeneity: crack, quasicrack, or anticrack, various types of boundary value problems are encountered and a unified approach using the existing mathematical treatments is not possible, so that researchers have employed different methods. For the cases of in-plane and out-of-plane strain, and under a uniform applied far-field loading, Hurtado et al. [22] show that the lamellar inhomogeneity can equivalently be replaced by a suitable distribution of dislocations, which can be obtained by using the concept of surface dislocation density. Also, Homentcovschi and Dascalu [23] use the Muskhelishvili's complex potentials to study the two-dimensional problems of lamellar inhomogeneity in unbounded isotropic elastic materials. The deficiency of the two latter approaches is the difficulty in their extension, not only to three-dimensional lamellar inhomogeneity, but also for remote polynomial loading. On the other hand, the beauty of the Eshelby's [24–26] result is that there is an exact correspondence between the far-field loading and the form of the distribution of homogenizing eigenstrain inside the equivalent inclusion. This powerful theory implies many dramatic results. In the context of the present study, as one of its abundant applications, it enables us to find the exact SIFs for the two- and three-dimensional lamellar inhomogeneities due to a far-field applied polynomial loading.

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