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Finite element analysis of piezoelectric corner configurations and cracks accounting for different electrical permeabilities

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Abstract

Based on eigenfunctions of asymptotic singular electro-elastic fields obtained from a kind of *ad hoc* finite element method [Chen MC, Zhu JJ, Sze KY. Finite element analysis of piezoelectric elasticity with singular inplane electroelastic fields. Engng Fract Mech 2006;73(7):855–68], a super corner-tip element model is established from the generalized Hellinger–Reissner variational functional and then incorporated into the regular hybrid-stress finite element to determine the coefficients of asymptotic singular electro-elastic fields near a corner-tip. The focus of this paper is not to discuss the well-known behavior of electrically impermeable and permeable (usually it means fully permeable, hereinafter the same) cracks but analyze the limited permeable crack-like corner configurations embedded in the piezoelectric materials, i.e., study the influence of a dielectric medium inside the corner on the singular electro-elastic fields near the corner-tip. The boundary conditions of the impermeable or permeable corner can be considered as simple approximations representing upper and lower bounds for the electrical energy penetrating the corner. Benchmark examples on the piezoelectric crack problems show that present method yields satisfactory results with fewer elements than existing finite element methods do. As application, a piezoelectric corner configuration accounting for the limited permeable boundary condition is investigated, and it is found that the limited permeable assumption is necessary for corners with very small notch angles.

Keywords: Piezoelectric elasticity; Corner; Singular electro-elastic field; Finite element; Limited permeable

1. Introduction

Piezoelectric corner configurations with different notch angles (see Fig. 1) exist often in intelligent structures. Subsequently, electro-elastic field singularities exist around the corner-tip o. When the intelligent structures are exposed to internal or external mechanical and/or electric loadings, high mechanical and electric field concentrations may arise and lead to abnormal working states or structure failure. Therefore, the optimum design and reliable service lifetime predictions of the intelligent structures inevitably requires a thorough understanding of their fracture behavior under mechanical and electric loadings.

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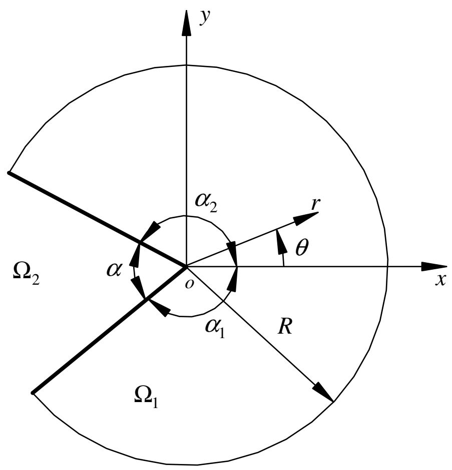

Fig. 1. The corner-tip domain of a piezoelectric corner configuration.

Since Parton analyzed the fracture mechanics of piezoelectric materials from a theoretical stand point of view in 1976 [2], great progresses have been made on the studies of piezoelectric crack problems. Until now, many researchers have reported their work [3–11]. However, in these studies there is a common deficiency that it is assumed that the permittivity of the dielectric medium interior to the crack is zero, in contrast to the reality that electric fields can permeate free space and any gas occupying it. Therefore, much attention has been paid on the electric boundary conditions such as limited permeable, permeable and conducting [12–18].

Even though a large amount of work dealing with the piezoelectric crack problems has been carried out as mentioned above, comparatively, studies on the piezoelectric fracture mechanics of corner-tip are rare. There are a few exploring investigations on piezoelectric corner configurations accounting for different electrical permeabilities [19–21], but these studies are only focused on the discussions of the orders of electro-elastic singularities. Recently, Scherzer and Kuna [22] analyzed interface corner and crack configurations embedded in smart composite materials. However, in their paper the electrical boundary condition inside the corners was still restricted to impermeable one. We have not been aware of any reports on the electro-elastic fields near the corner-tips of piezoelectric corner configurations accounting for the limited permeable boundary conditions.

2. *Ad hoc* finite element formulation for sectorial domains

For the piezoelectric corner configuration, the limited permeable boundary condition is given as

$$\left(D_\theta^{\text{piezo}} - D_\theta^{\text{medium}}\right)|_{\theta=\alpha_1} = 0, \quad \left(D_\theta^{\text{piezo}} - D_\theta^{\text{medium}}\right)|_{\theta=\alpha_2} = 0 \tag{1}$$

in which superscripts 'piezo' and 'medium' denote the components in the piezoelectric corner configuration and the permeable medium, respectively. D_θ is the hoop electric displacement. α_1 and α_2 denote the corner angles (see Fig. 1). Obviously, the influence of corner angle of the magnitude α has been included in Eq. (1). By taken Eq. (1) as the precondition, an *ad hoc* finite element formulation can be developed to discretize the corner-tip domain composed of piezoelectric material and dielectric medium. The finite element formulation is written as [1]

$$\sum_{e^{\text{piezo}}} \left[\begin{Bmatrix} \delta\mathbf{q}_\text{m}^\text{e} \\ \delta\mathbf{q}_\text{e}^\text{e} \end{Bmatrix}^\text{T} (\lambda^2\overline{\mathbf{P}}^\text{e} + \lambda\overline{\mathbf{Q}}^\text{e} + \overline{\mathbf{R}}^\text{e}) \begin{Bmatrix} \mathbf{q}_\text{m}^\text{e} \\ \mathbf{q}_\text{e}^\text{e} \end{Bmatrix} \right] + \sum_{e^{\text{medium}}} \left[\left(\delta\mathbf{q}_\text{e}^\text{e}\right)^\text{T} (\lambda^2\mathbf{P}^\text{e} + \lambda\mathbf{Q}^\text{e} + \mathbf{R}^\text{e})\mathbf{q}_\text{e}^\text{e} \right] = \mathbf{0} \tag{2}$$

in which 'e' denotes the element components; $\mathbf{q}_\text{m}^\text{e}$ and $\mathbf{q}_\text{e}^\text{e}$ are vectors including the nodal displacement and potential components respectively; Matrices $\overline{\mathbf{P}}^\text{e}$, $\overline{\mathbf{Q}}^\text{e}$, $\overline{\mathbf{R}}^\text{e}$, $\mathbf{P}^\text{e}$, $\mathbf{Q}^\text{e}$ and $\mathbf{R}^\text{e}$ are defined as follows: