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An effective medium model for elastic waves in microcrack damaged media

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ABSTRACT

Direct numerical simulations of waves traveling through microcrack-damaged media are conducted and the results are compared to effective medium calculations to determine the applicability of the latter for studying wave propagation. Both tensile and compressive waves and various angular distributions of randomly-located cracks are considered. The relationships between the input wavelength and the output wave speed and output signal strength are studied. The numerical simulations show that the wave speed is nearly constant when $1/ka > 60$ for tensile waves and $1/ka > 10$ for compressive waves, where k is the wave number and a is the average half-crack length. The direct simulations also show that when the input wavelength is much longer than the crack length, $1/ka > 60$, the wave can pass through the damaged medium relatively unattenuated. On the other hand, when the input wavelength is shorter than a "cut off" wave length, the output wave magnitude decreases linearly with the input wavelength. The effective medium wave speed and magnitude calculations are not dependent on the input wavelength and therefore the results correspond well with the numerical simulations for large $1/ka$. This suggests a minimum wavelength for which the homogenized methods can be used for studying these problems.

1. Introduction

For many brittle materials, the most common damage mechanism is microcracking. As microcracks develop, they change the local mechanical response to input loads. One way, commonly used to evaluate the quasi-static response of the damaged medium is by directly simulating an actual array of cracks. Solutions of this type include the fast multipole method (FMM, [1,2]), boundary element method (BEM [3,4]) and many other numerical solutions, such as described in [5–9]. The major advantage of these methods is that they take into account the actual crack–crack interactions. There is no theoretical limitation to the crack density or crack distribution and these direct methods can give highly accurate results if the model is an accurate representation of the damaged medium. However, the detailed calculation of the interaction between the cracks is also the major disadvantage of the direct methods. As the crack density increases, the modeling time and computational demands can increase dramatically.

To address this increase in computational demand, effective medium methods can be used. In these methods, one models the quasi-static response of a medium with diffuse microcrack damage by replacing the damaged material with an effective medium, possessing the same local mechanical properties. Many such effective medium models have been developed, such as the self consistent method (SCM) [10–12], Generalized self consistent method (GSCM) [13–17], Mori–Tanaka method (MTM) [18,19], and the differential scheme method (DSM) [20]. For a comprehensive review of the literature in effective mechanical properties prediction, readers can refer to [6,21].

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While the above-cited literature primarily discusses the quasi-static response of microcrack-damaged media, several researchers have addressed elastic wave propagation behavior in these materials. In 1995 Sayers and Kachanov [22] used the effective moduli of a cracked medium to study microcrack-induced elastic wave anisotropy. In that paper, the authors used second-rank and fourth-rank crack density tensors to evaluate the effective elastic moduli. They calculated the effective moduli for the (randomly-oriented) isotropic distribution of cracks and the perfectly aligned cracks. For the isotropic cracked medium, they also back-calculated the crack density from experimental data of the ultrasonic wave speed. Following Sayers and Kachanov's results, Schubnel and Gueguen [23,24] used micromechanical and statistical physics to evaluate the elastic wave velocities and permeability of cracked rocks. They calculated the velocity for high and low frequency waves for both wet and dry cracks for an aligned array of cracks and an array composed of ±60° cracks under an external confining pressure. Maurel et al. [25] studied the elastic wave propagation through a random array of dislocations. Markov [26] used the Frenkel–Biot theory to study the longitudinal harmonic wave propagation in an isotropic porous matrix with inclusions. Levin and Markov [27] used an effective medium approximation (EMA), based on the self-consistent method, to determine the effective elastic moduli and elastic wave propagation velocities in a transversely isotropic solid containing aligned inclusions.

Yet, with all this literature, there are several questions which remain. For example, under what conditions can we model an actual damaged material with an equivalent effective medium? How accurate is the effective medium representation? What factors affect the accuracy?

Several researchers have performed studies to address some of these topics. Zhang and Gross [28] studied the propagation of an elastic wave through a medium with randomly distributed cracks. The cracks are treated as finite length lines with displacement discontinuity and the crack surfaces are assumed to be frictionless. They studied the scattering function for a single crack and expanded this using a numerical approach to study the response of multiple cracks. They used both the theory of Foldy and the causal approach based on K–K relations to compute the attenuation coefficient and the effective wave velocity. Littles et al. [29] performed an experimental and theoretical investigation to study how longitudinal waves are scattered by a distribution of cracks. The results show that the transmission coefficients are a function of the incident wave number, the crack size and crack density. Kawahara and Yamashita [30] studied the scattering of P, SV and SH waves by a zonal distribution of uniaxial cracks using a theoretical analysis. They showed that the attenuation coefficient peaks around $ka \approx 1$, the phase velocity is almost independent of k in the range of $ka < 1$ and increases monotonically when k is in the range of $ka > 1$. They also found that the effect of crack-face friction is to shift the peak of the attenuation coefficient toward the lower wave number range. Kelner et al. [31] used the boundary element method to simulate P wave propagation in media with uniaxial cracks. The authors conclude that when the wavelength of the incident wave is close to, or shorter than, the crack length, the scattered waves are efficiently excited. They also observed that the attenuation factor of the direct P wave peaks at around $ka = 2$, where k is the incident wave number and a is the crack length, and decreases proportionally with $(ka)^{-1}$ in the high wave number range.

Most of the above papers focus on low crack densities and high frequency input waves, a combination which corresponds to high attenuation. Also, the crack orientations in most of the papers are limited to aligned or randomly-oriented distributions. Additionally, in most of the literature cited, the authors assume the crack surfaces are stress free, which means the cracks are opened under all loads. In our previous paper [16], we use the generalized self consistent method (GSCM) to predict the anisotropic effective moduli of a medium containing an arbitrarily-oriented distribution of cracks. In that paper, we show that because the cracks open or close under different external loads, the effective moduli under different loading conditions are quite different. In that paper, we discussed three different loading conditions: (1) overall applied tensile loads; (2) overall compressive loads; (3) initially stress free media. The effective moduli under these three different loading conditions are evaluated taking into account the effects of crack-face contact. In that paper we also mentioned that tensile and compressive wave propagation in the same medium behave very differently. In this paper, we study under what conditions one may use the calculated effective moduli to simulate wave propagation in the microcrack damaged medium. We conduct direct numerical simulations to relate input wavelength to propagating wave speed and output signal strength, and compare these results to those obtained via the effective medium models. Once these comparisons are established, we can establish guidelines for the use of effective medium calculations to analyze ultrasound data for determining the internal state of microcrack damage.

2. FEM model

In the following numerical experiments, we use the definition of crack density and crack orientation distribution function $\phi(\theta)$ described in [15,16]. We define the crack density as

$$\eta = \frac{Ma^2}{A}, \tag{1}$$

where M is the number of cracks per unit area A, and a is the average half-crack length. We assume the crack orientation can be described by an orientation distribution function $\phi(\theta)$, where θ is the angle of the individual crack with respect to the average crack orientation, assigned to be the x-axis. In the following analysis, we assume that the cracks are evenly distributed between the angles $-\theta_0$ and $+\theta_0$, as shown in Fig. 1. A more detailed description of the crack distribution function $\phi(\theta)$, and crack density function η, can be found in [16].