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Note

Engineering approach to penetration modeling

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ABSTRACT

We suggest an approach that allows deriving penetration equations using semi-empirical analytical dependence between the impact velocity and the depth of penetration (DOP). The application of the suggested method is demonstrated for the widely used Young's models which describe penetration in the main types of soils.

1. Introduction

Approximate methods are widely used in penetration mechanics as can be seen from inspecting the list of references in a recently published monograph [1]. The latter can be supplemented by the studies [2–6] which include also overviews of the relevant investigations. Literature on penetration mechanics contains a number of particular examples of the engineering approach that allows to derive the penetration equations using semi-empirical or empirical dependence between the impact velocity and the depth of penetration (DOP) [7,8]. The main goals of this Note are: (i) attract attention to this useful method, (ii) formulate the method in a general form; (iii) illustrate the application of this method on the example of the widely used system of Young's models which describe penetration in the main types of soils.

2. Investigation of the problem

Consider a normal penetration of a rigid body into a semi-infinite shield along the axis h, where the coordinate h, the instantaneous location of the penetration, is defined as the distance between the leading edge of the impactor and the front surface of the shield; the effects associated with the stage of the incomplete immersion of the impactor's nose in the shield are neglected.

Assume that the empirical dependence, $h_{max} = P(v_{imp})$, between the impact (initial) velocity, v_{imp}, and the DOP, h_{max}, is known. The problem is to determine a function G assumed to be dependent on velocity of the impactor, v, which determines the total force acting at the impactor. Clearly, the functions P and G depend also on the parameters characterizing the mechanical properties of the shield and the shape of the impactor which are not listed as arguments.

The equation describing the motion of the impactor with mass m can be written as follows:

$$mv\,dv/dh = -G(v). \tag{1}$$

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The solution of Eq. (1) with the initial condition $v(h_0) = v_{imp}$ (h_0 will be defined below) reads:

$$h = h_0 - m\int_{v_{imp}}^{v} \Theta(V)\,dV, \quad \Theta(V) = \frac{V}{G(V)}. \tag{2}$$

Substituting $v = 0$ and $h = h_{max}$ into Eq. (2) we obtain the relationship between the DOP and the impact velocity that must recover the given empirical dependence, $h_{max} = P(v_{imp})$. Consequently we obtain

$$P(v) = h_0 + m\int_0^{v} \Theta(V)\,dV, \tag{3}$$

where the variable v_{imp} is replaced by v. Eq. (3) is an integral equation with respect to the function $\Theta(V)$. The unique and continuous solution of this equation

$$\Theta(V) = P'(V)/m, \tag{4}$$

which is of our interest, exists if

$$P(0) = h_0 \tag{5}$$

and $P(V)$ and $P'(V)$ are continuous functions [9]. Eqs. (2) and (3) imply the following expression for unknown function G:

$$G(v) = mv/P'(v). \tag{6}$$

Here we assumed that $0 \leqslant \lim_{v\to 0} v/P'(v) < +\infty$, and $P'(v) > 0$ for $v > 0$ because for physical reasons $P(v)$ must be an increasing function.

Substituting Eq. (6) into Eq. (2) we obtain

$$h(v) = P(0) + P(v_{imp}) - P(v). \tag{7}$$

Since $dt = dh/v = (dh/dv)(dv/v) = -P'(v)\,dv/v$, where t is time, then

$$t(v) = -\int_{v_{imp}}^{v} P'(V)\,dV/V = \chi(v_{imp}) - \chi(v), \quad \chi(w) = \int_0^{w} P'(V)\,dV/V. \tag{8}$$

Eqs. (7) and (8) imply the equation of motion of the impactor in parametrical form whereby the decrease of the parameter v, varying from v_{imp} to 0, is associated with penetration of the impactor from the initial moment until the stop. Clearly, the time of penetration is $t(0)$.

The major impediment to the direct application of this method is the requirement that the first derivative $P'(v)$ must be a continuous function. Indeed, numerous engineering models employ the relationship between the DOP and the impact velocity which is determined by different formulas for particular intervals of the impact velocity v_{imp}. Clearly, at the end points of these intervals the derivative $P'(v)$ is discontinuous. The discontinuities are generally not associated with the physics of penetration, and the dependence $h_{max} = P(v_{imp})$ can be smoothed without impairing the performance of the model. Let us validate the latter claim using as the example the system of Young's models which describe penetration in the main types of soils.

3. Application to the Young's models

The widely used Young's penetration models for soil, rock and concrete (SRC) shields as well as for ice and frozen soil (IFS) [10] can be written in the following form:

$$\frac{P(v_{imp})}{k} = \tilde{P}(v_{imp}) = \begin{cases} P_a(v_{imp}) & \text{if } v_{imp} < v_* \\ P_b(v_{imp}) & \text{if } v_{imp} \geqslant v_* \end{cases}, \tag{9}$$

where

$$P_a(v_{imp}) = \alpha_1 \ln\left(1 + \alpha_2 v_{imp}^2\right), \quad P_b(v_{imp}) = k_1(v_{imp} - v_0), \tag{10}$$

k is a known function depending on the mass and shape of the impactor and on the mechanical properties of the shield, P is the DOP in meters, v_{imp} is the impact velocity in m/s, $v_0 = 30.5$ m/s, $v_* = 2v_0 = 61$ m/s, $\alpha_1 = 0.0008$ (0.00024), $\alpha_2 = 0.000215\ c^2/\text{m}^2$, $k_1 = 0.000018$ (0.0000046) for SRC (IFS). We use this model in the original form [10] with the dimensional coefficients.

Young's dependence between P/k and v_{imp} for SRC is shown in Fig. 1. It is immediately obvious that the function is discontinuous in the point $v_{imp} = v^*$, $P_a(v^*) = 0.000470$, $P_b(v^*) = 0.000549$. This discontinuity has no physical meaning and only causes problems when applying these relations.

Let us modify the model for the relatively small v_{imp} in order to construct a smooth approximation of the function $\tilde{P}(v_{imp})$. The problem is to find such $\alpha_1 > 0$, $\alpha_2 > 0$ and v^* that

$$P_a(v_*) = P_b(v_*), \quad P'_a(v_*) = P'_b(v_*). \tag{11}$$