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Transient thermoelectroelastic response of a functionally graded piezoelectric strip with a penny-shaped crack

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Abstract

Transient response of a penny-shaped crack in a plate of a functionally graded piezoelectric material (FGPM) is studied under thermal shock loading conditions. It is assumed that the thermoelectroelastic properties of the strip vary continuously along the thickness of the strip, and that the crack faces are completely insulated. By using both the Laplace and Hankel transforms, the thermal and electromechanical problems are reduced to a singular integral equation and a system of singular integral equations which are solved numerically. The intensity factors vs. time for various crack size, crack position and material nonhomogeneity are obtained.

Keywords: Electroelasticity; Fracture mechanics; Functionally graded piezoelectric strip; Penny-shaped crack; Thermal shock; Integral transform

1. Introduction

Due to the rapid growth in application for smart or intelligent systems [1–5], the mechanical and fracture properties of piezoelectric ceramics under thermal loading conditions are becoming more and more important. While so much attention has been focused on thermoelectromechanical fracture analyses of homogeneous piezoelectric materials with a two-dimensional crack [6–10], and with a penny-shaped crack [11], there are still very few articles considering the fracture problems in functionally graded piezoelectric materials (FGPMs) [12] under thermal loading [13,14]. Thus, the present author took into consideration a study of the steady mixed-mode thermoelectromechanical fracture problem for an FGPM strip under thermal loading with a two-dimensional crack [15,16] and with a penny-shaped crack [17].

Additionally the overshoot phenomenon of the intensity factors is also observed in a homogeneous piezoelectric plate under thermal shock loading [18], and this overshoot phenomenon is more remarkable in an FGPM strip with a two-dimensional crack [19]. Thus the author also considered it important to investigate the transient thermal fracture behavior of an FGPM strip with a penny-shaped crack.

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Nomenclature

c crack radius
$c_{kl}(z)$, c_{kl0} elastic stiffness constants at z and $z = 0$
$c_p(z)\rho(z)$, $c_{p0}\rho_0$ heat capacity at z and $z = 0$
$D_{ri}(r,z,t)$, $D_{zi}(r,z,t)$ electric displacement components
$D_{z0}(r,t)$ thermally induced electric displacement component
$e_{kl}(z)$, e_{kl0} piezoelectric constants at z and $z = 0$
F normalized time
h thickness of the strip
h_1 thickness of the strip in the region of $0 \leqslant z$
h_2 thickness of the strip in the region of $z \leqslant 0$
$H(t)$ Heaviside step function of time
K_{I}, K_{II} stress intensity factors
K_{D} electric displacement intensity factor
p Laplace parameter
$p_3(z)$, p_{30} pyroelectric constant at z and $z = 0$
r, θ, z cylindrical coordinates
$T_i(r,z,t)$ temperatures
$T_D(t)$ temperature difference across the crack surfaces at $r = 0.0$
t time
T_0 uniform temperature change at $z = h_1$
T_{I} initial temperature
$u_{ri}(r,z,t)$, $u_{zi}(r,z,t)$ displacement components
β, ω, δ positive or negative constants (nonhomogeneous parameters)
$\varepsilon_{kk}(z)$, ε_{kk0} dielectric constants at z and $z = 0$
$\kappa_k(z)$, κ_{k0} coefficients of heat conduction at z and $z = 0$
λ_0 thermal diffusivity
$\lambda_{kk}(z)$, λ_{kk0} stress–temperature coefficients at z and $z = 0$
$\sigma_{rri}(r,z,t)$, $\sigma_{\theta\theta i}(r,z,t)$, $\sigma_{zzi}(r,z,t)$, $\sigma_{zri}(r,z,t)$ stress components
$\sigma_{zz0}(r,t)$, $\sigma_{zr0}(r,t)$ thermally induced stress components
$\phi_i(r,z,t)$ electric potentials

In this paper, a penny-shaped crack in a plate of a functionally graded piezoelectric material mathematically modeled by a nonhomogeneous solid is studied under thermal shock loading conditions. The medium is initially at a uniform temperature T_{I} and is suddenly subjected to a uniform temperature rise T_0 along one of the traction-free boundaries. It is assumed that material properties depend only on the coordinate z (perpendicular to the crack faces) in such a way that the properties show some exponential functions of z. By using both the Laplace and Hankel transforms [20], the thermal and electromechanical problems are reduced to a singular integral equation and a system of singular integral equations [21]. The singular integral equations are solved numerically [22], and a numerical method is then employed to obtain the time dependent solutions by way of a Laplace inversion technique [23]. Numerical calculations are carried out, and detailed results are presented to illustrate the influence of the crack size, crack location and material nonhomogeneity on the time dependences of the stress and electric displacement intensity factors.

2. Formulation of the problem

As shown in Fig. 1, we can consider an axially symmetric problem. A penny-shaped crack of radius c is embedded in a functionally graded piezoelectric strip of thickness $h = h_1 + h_2$. The crack is located parallel to the boundaries and at an arbitrary position in the strip. The cylindrical coordinate system is denoted by