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# Elastic–plastic fracture mechanics analyses of circumferential through-wall cracks between elbows and pipes

Chang-Kyun Oh [a], Tae-Kwang Song [b], Yun-Jae Kim [b,*], Jong-Sung Kim [a], Tae-Eun Jin [a]

[a] *Structural Integrity and Materials Department, Korea Power Engineering Company, Yongin-si, Gyeonggi-do 449-713, Republic of Korea*

[b] *Department of Mechanical Engineering, Korea University, 1–5 Ka, Anam-Dong, Sungbuk-Ku, Seoul 136-701, Republic of Korea*

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**Abstract**

The present work proposes a method for elastic–plastic fracture mechanics analysis of the circumferential through-wall crack in weldment joining elbows and attached straight pipes, subject to in-plane bending. Heterogeneous nature of weldment is not explicitly considered and thus, the proposed method assumes cracks in homogeneous materials. Based on small strain finite element limit analyses using elastic–perfectly plastic materials, closed-form limit loads for circumferential through-wall cracks between elbows and straight pipes under bending are given. Then applicability of the reference stress-based method to approximately estimate $J$ and crack opening displacement (COD) is evaluated. It was found that the limit moments for circumferential cracks between elbows and attached straight pipes can be much lower than those for cracks in straight pipes, particularly for a crack length of less than 30% of the circumference; this result is of great interest in practical cases. This result implies that, if one assumes that the crack locates in the straight pipe, limit moments could be overestimated significantly, and accordingly, reference stress-based $J$ and COD could be significantly overestimated. For the leak-before-break analysis, accurate $J$ and COD estimation equations based on the reference stress approach are proposed.


*Keywords:* Circumferential through-wall crack; Crack opening displacement; Elbow; Finite element analysis; *J*-integral; Leak-before-break; Reference stress method

## 1. Introduction

Application of the leak-before-break (LBB) concept [1,2] to design and integrity analyses of nuclear piping is important. The underlying idea of the LBB concept is that the materials used in nuclear piping are sufficiently tough that even a large circumferential through-wall crack, which could result in coolant leaking

* Corresponding author. Tel.: +82 2 3290 3372; fax: +82 2 926 9290.
*E-mail address:* kimy0308@korea.ac.kr (Y.-J. Kim).

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**Nomenclature**

| | |
|---|---|
| $E$ | Young's modulus |
| $F$ | non-dimensional function for elastic stress intensity factor |
| $J$ | $J$-integral |
| $J_e$ | elastically calculated $J$, see Eq. (10) |
| $K$ | elastic stress intensity factor |
| $M$ | moment |
| $M_L$ | limit moment of a cracked component |
| $M_0$ | limit moment of an un-cracked elbow |
| $M_{OR}$ | optimised reference moment for the reference stress |
| $M_0^s$ | limit moment of an un-cracked straight pipe |
| $n$ | strain hardening index ($1 \leqslant n < \infty$) for Ramberg–Osgood model, Eq. (9) |
| $R$ | bend radius |
| $r_m$ | mean pipe radius |
| $t$ | thickness of a pipe |
| $V$ | non-dimensional function for elastic crack opening displacement |
| $\alpha$ | coefficient of Ramberg–Osgood model, Eq. (9) |
| $\delta$ | elastic–plastic crack opening displacement (COD) |
| $\delta_e$ | elastically calculated crack opening displacement(COD) |
| $\varepsilon$ | strain |
| $\varepsilon_{ref}$ | reference strain |
| $\lambda$ | bend characteristic, $= Rt/r_m^2$ |
| $\nu$ | Poisson's ratio |
| $\theta$ | half circumferential angle of a circumferential crack |
| $\sigma_0$ | limiting stress of a perfectly plastic material; 0.2% proof (yield) stress |
| $\sigma_{ref}$ | reference stress |

*Abbreviations*

| | |
|---|---|
| LBB | leak-before-break |
| FE | finite element |
| COD | crack opening displacement |

rates well in excess of those detectable by leak detection systems, would still remain stable and thus, not result in a double-ended guillotine break under a maximum loading condition. One important element in applying the LBB concept to nuclear piping is the elastic–plastic fracture mechanics analysis for pipes with postulated circumferential through-wall cracks. Although extensive works have been done for developing elastic–plastic fracture mechanics analysis methods for circumferential through-wall cracked pipes (see Refs. [3–12]), they are mainly for straight pipes with circumferential through-wall cracks. More recently, Chattopadhyay et al. [13] proposed elastic–plastic $J$ and crack opening displacement (COD) estimation schemes for circumferential through-wall cracked elbows under in-plane bending, which can be used for the LBB analysis of postulated circumferential through-wall cracks in the centre of elbows.

In power plants, piping systems often involve elbows welded to straight pipes. Because welded regions are vulnerable to cracking, the LBB analysis needs to be performed for postulated circumferential through-wall cracks in weldments joining elbows and attached straight pipes, and thus requires an engineering method for elastic–plastic fracture mechanics analysis. Such a method has not been explored in the literature. For a circumferential through-wall crack between an elbow and attached straight pipe, one could argue that elastic–plastic fracture mechanics analysis could be performed assuming that the circumferential through-wall crack is located in the straight pipe. However, it will be shown in this paper that such an approach can lead to significantly different results, and thus should not be recommended.