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Flow characteristics of hydrogen gas through a critical nozzle

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ABSTRACT

Critical nozzles are widely used in the flow measurement and can be used for mass flow-rate measurement of hydrogen gas. The effect of real gas state equation on discharge coefficient of hydrogen gas flow through a critical nozzle was investigated. The real gas critical flow factor was introduced which considers the effect of the real gas on discharge coefficient. An analytic solution of real gas critical flow factor of hydrogen gas calculated from the modern equations of state based on Helmholtz energy, over a wider range of temperature 150–600 K and pressure up to 100 MPa was presented. An accurate empirical equation for real gas critical flow factor was determined by the nonlinear regression analysis. The equation was in good agreement with the high-pressure hydrogen gas experimental data by Morioka and CFD solutions by Nagao and Kim. Using this equation, the discharge coefficient can be directly and accurately calculated. It indicates that the discharge coefficient of hydrogen gas should be comprehensively taken into consideration with stagnation temperature, stagnation pressure and nozzle throat diameter. A lot of detailed results about the effect of real gas state equation were obtained.

1. Introduction

It is known that hydrogen gas is one of the most promising gases as a future alternative and renewable energy source [1,2]. In such an application, a precise measurement of hydrogen mass flow-rate is very important for mileage and power output of the vehicle [3].

Critical nozzles are widely used in the flow measurement as flow meters and can be used for mass flow-rate measurement of hydrogen gas, since the flow measurement process of sonic nozzle is not affected by its downstream flow disturbance or pressure fluctuation [4,5]. Such as, Morioka et al. [6] had developed a critical nozzle flow meter for high-pressure hydrogen gas flow measurements, and its characteristics were experimentally examined with hydrogen gas pressure up to 70 MPa.

Due to the complex relationships between mass flow-rate through a critical nozzle and thermophysical properties of hydrogen gas, few studied has been reported on the discharge coefficient of hydrogen gas flow through a critical nozzle. Johnson [7] utilized the compressibility factor equation of hydrogen gas to research the variable of hydrogen mass flow-rate though a critical nozzle. Kim et al. [8,9] investigated the real gas effect of high-pressure and high Reynolds number hydrogen gas through a critical nozzle by using a CFD method. It indicated that the discharge coefficient in the range of high-pressure decreases with an increase of Reynolds number because of the effect of real gas. Nagao et al. [10] presented by CFD method that the discharge coefficient of real gas for N_2,

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Nomenclature

A	Area, m^2
a	Helmholtz energy, J/kg
C_d^i	ideal discharge coefficient, –
C_d^r	real discharge coefficient, –
c_v	Isochoric specific heat capacity, J/(kg K)
C_i^*	ideal gas critical flow factor, –
C_r^*	real gas critical flow factor, –
d	throat diameter of nozzle, mm
h	specific enthalpy, J/kg
M	molar mass, kg/mol
q_m	real gas mass flowrate, kg/s
q_{mi}	ideal gas mass flowrate, kg/s
p	absolute pressure, Pa
R	universal gas constant, 8.31451 J/(mol K)
Re	Reynolds number, –
R_m	specific gas constant, R/M, J/(kg K)
s	specific entropy, J/(kg K)
T	absolute temperature, K
u	velocity, m/s
v	specific volume, m^3/kg
w	speed of sound, m/s
Z	compressibility factor, –

Greek

α	reduced Helmholtz energy, –
δ	reduced density, –
γ	isentropic exponent, –
φ	multiplier of iteration convergence, –
κ	isothermal bulk modulus, ($\kappa = -v(\partial p/\partial v)_T$, Pa
μ	dynamic viscosity, Pa s
π	reduced pressure, p/p_c
ρ	density, kg/m^3
σ	isobaric coefficient of cubical expansion,(σ =(1/ v)($\partial v/\partial T$)$_p$, 1/K
τ	inverse reduced temperature, T_c/T

Superscripts

i	ideal gas
o	ideal-gas property
r	residual/real gas

Subscripts

0	at stagnation conditions
t	at nozzle throat conditions
c	critical point of Hydrogen

CH_4 and CO_2 decreases with an increase of Reynolds number. And then, they [3] investigated the effect of state equation on the discharge coefficient and thermodynamic properties through the critical nozzle by using H_2 with the help of CFD method by Redlich–Kwong [11], Lee-Kesler [12] and Peng–Robinson equations of state [13]. However, most of previous studies were lack of many details knowledge and understanding of the effect of real gas state equation and had not established the calculating formula for hydrogen mass flowrate though a critical nozzle.

In this paper, an accurate analytic equation of discharge coefficient of hydrogen gas flow through a critical nozzle was present. This equation was validated using experimental data by Morioka [6] and CFD solutions by Nagao [3] and Kim [9] for critical nozzle flow of hydrogen gas. Lots of new results about the effect of real gas state equation on discharge coefficient were obtained.

2. Critical flow calculations for hydrogen gas

For one-dimensional isentropic flow of ideal gas, the ideal critical flow-rate q_{mi} is calculated by the following equation [5]

$$q_{mi} = \frac{A_t C_i^* p_0}{\sqrt{R_m T_0}} \tag{1}$$

where, ideal gas critical flow factor C_i^* is equal to $\sqrt{\gamma(2/\gamma+1)^{\gamma+1/\gamma-1}}$. For the Hydrogen, C_i^* is constant with a value of 0.687473. In practical, the real gas flow-rate is different from Eq. (1) because of the effect of viscous, multidimensional and real gas state equation. The definition of the ideal discharge coefficient which is usually simplified as “discharge coefficient” can be calculated by

$$C_d^i = \frac{q_m}{q_{mi}} = \frac{q_m\sqrt{R_m T_0}}{A_t C_i^* p_0} \tag{2}$$

Replacing the ideal gas critical flow factor C_i^* with the real gas critical flow factor C_r^*, the real discharge coefficient C_d^r is defined by

$$C_d^r = \frac{q_m\sqrt{R_m T_0}}{A_t C_r^* p_0} = \left(\frac{C_i^*}{C_r^*}\right) C_d^i \tag{3}$$

where, C_r^* is real gas critical flow factor the relative which consider the effect of the real gas on flow-rate. The real discharge coefficient C_d^r is preferred over the ideal discharge coefficient because it is always less than unity and independent of the real gas effects. Thus, the real discharge coefficient equations of different gases will be the same [14].

The real discharge coefficient C_d^r is affected by viscous and geometric. For the ISO 9300 sonic nozzle, the real discharge coefficient can be described by Ref. [5]

$$C_d^r = 0.9985 - 3.412 Re_t^{-0.5}, \quad 2.1\times10^4 < Re_t < 1.4\times10^6 \tag{4}$$

where,

$$Re_t = \frac{4q_m}{\pi d \mu_0} \tag{5}$$

The discharge coefficient C_d^i can be divided into two parts, namely, C_d^r which consider the effect of viscous and multidimensional and C_r^*/C_i^* which only consider the effect of real gas.

To obtain the relative critical flow factor C_r^*/C_i^*, it is necessary to determine the value of real gas critical flow factor C_r^* which can be calculated by

$$C_r^* = \rho_t u_t \frac{\sqrt{R_m T_0}}{p_0} \tag{6}$$