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Order reduction via balancing and suboptimal control of a fuel cell – Reformer system

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ABSTRACT

In this paper we first perform system balancing of an eighth-order mathematical model of a polymer electrolyte membrane fuel cell (PEMFC) dynamic *coupled* with a tenth-order mathematical model of a hydrogen gas reformer. Based on that information we determine reduced-order mathematical models of the original eighteen-order model by eliminating state variables that have negligible contribution to the model dynamics. Having obtained the reduced-order models, we study their step and impulse responses, and compare them to those of the original full-order model. In addition, we design corresponding suboptimal feedback controllers based on the reduced-order models. Comparing the obtained suboptimal controllers (that require a reduced number (only six or even five) of feedback loops making them easy for implementation) we find that their suboptimal performances are very close to the optimal performance of the full-state optimal feedback controller. It is important to emphasize that the full-order state feedback controller requires the same number of the feedback loops as the dimension of the original full-order state space model (in this case eighteen), which makes it complex and sometimes impractical to implement.

1. Introduction

Modeling and control of polymer electrolyte membrane or proton exchange membrane fuel cells (PEMFC) has become a modern research trend both theoretically and experimentally; see Refs. [1–18] and references therein. Adaptive control of PEM fuel cells was presented in Ref. [1]. Modeling and control of PEM fuel cells was considered in Refs. [1,7,11,12,16,17]. Studies of sliding mode control for PEM fuel cells can be found in Refs. [4,8,9]. Slow and fast dynamics of a hydrogen gas reformer were studied in Ref. [10]. The hydrogen gas reformer which produces hydrogen from hydrogen rich fuels has been recently included in several modeling and control system studies, either as an independent system or coupled with a fuel cell system [14,17,18]. The fuel cell and reformer mathematical models that comprise the coupled system discussed in this paper have been studied from different points of views independently in Refs. [11,12,18]. A fuzzy controller for a hydrogen–air fuel cell system was designed in Ref. [17]. Hydrogen production using reformers has been a modern research topic as demonstrated in recent journal papers [19–22]. In this paper, for the first time, the coupled PEM fuel cell and a hydrogen gas reformer are studied using the system balancing transformation and the corresponding system-order reduction technique for design of suboptimal controllers.

This paper is organized as follows. In Section 2 we have presented mathematical models of PEM fuel cell, hydrogen

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Nomenclature

m_{O_2}	mass of oxygen
m_{H_2}	mass of hydrogen
m_{N_2}	mass of nitrogen
ω_{cp}	compressor speed, rad/sec
p_{sm}	pressure of gas in supply manifold
m_{sm}	mass of gas in supply manifold
$m_{H_2O_A}$	mass of water in the anode channel
p_{rm}	pressure in the return manifold
W_{cp}	compressor flow rate
v_{st}	stack voltage
v_{cm}	compressor motor input voltage
I_{st}	stack current
u_{blo}	air blower signal
u_{valve}	valve blower signal
T_{cpox}	catalyst temperature
$p_{H_2}^{an}$	pressure of hydrogen in the anode
p^{an}	anode pressure
p^{hex}	heat exchanger pressure
ω_{blo}	speed of the blower, rad/sec
p^{hds}	pressure of hydro-desulfurizer
$p_{CH_4}^{mix}$	pressure of CH_4 in the mixer
p_{air}^{mix}	pressure of air in the mixer
$p_{H_2}^{wrox}$	hydrogen pressure in water gas shift converter (WROX)
p^{wrox}	total pressure in WROX

gas reformer, and linearized model of their coupling. Section 3 reviews the importance of observability, controllability, and stability concepts and their use in system model order reduction via the system balancing transformation. In Section 4, we have studied step and impulse responses of the obtained reduced order models and compared them to the exact (full-order) model. A suboptimal controller obtained using the reduced order models and corresponding number of reduced-order feedback loops is defined and designed in Section 5, where its performance is compared also to the performance of the optimal controller that requires a large number of feedback loops (eighteen).

2. Model description

In this section we will first present the mathematical models of PEM fuel cell and hydrogen gas reformer, and then joint them into a coupled system and present the corresponding augmented mathematical model.

2.1. PEMFC mathematical model

The PEM fuel cell model considered in this paper was developed at the University of Michigan by Professor Stefanopoulou and her coworkers [11–14]. This 75 kW PEMFC has been used in a P2000 Ford electric car prototype. Schematics of the fuel cell system that includes the supply and return manifolds and the hydrogen tank is presented in Fig. 1. The developed mathematical model has been considered as one of the most comprehensive mathematical models of fuel cells. The *compressor* pumps air and it is modeled by a first-order differential equation for its angular velocity. Air is humidified via injection of water into the air stream, and the humidification process is modeled as static (by an algebraic equation). The *supply manifold* is modeled as a second-order dynamic system (using a second order differential equation) for pressure and gas mass. The *return manifold* is modeled as a first-order dynamic system for pressure in it. The *cathode-membrane-anode fuel cell dynamics* is modeled by a fifth order dynamic system with the following dynamics: pressures of hydrogen (H_2), oxygen (O_2), nitrogen (N_2), and water vapor on anode and cathode sides (respectively denoted by H_2O_A and H_2O_C). The tank pressurized at 10 atm supplies hydrogen.

The corresponding mathematical model obtained in Refs. [11,12] is represented by a system of nine nonlinear differential equations

$$\begin{aligned}\frac{dx^{FC}(t)}{dt} &= f_{FC}(x^{FC}(t), u^{FC}(t), w(t))\\ x^{FC} &= \left[x_1^{FC}\quad x_2^{FC}\quad x_3^{FC}\quad x_4^{FC}\quad x_5^{FC}\quad x_6^{FC}\quad x_7^{FC}\quad x_8^{FC}\quad x_9^{FC}\right]^T\\ &= \left[m_{O_2}\quad m_{H_2}\quad m_{N_2}\quad \omega_{cp}\quad p_{sm}\quad m_{sm}\quad m_{H_2O_A}\quad m_{H_2O_C}\quad p_{rm}\right]^T\end{aligned} \tag{1}$$

where subscripts *cp*, *sm*, *rm* stand for the compressor, supply manifold, and return manifold respectively. $\omega_{cp}(t)$ denotes the angular velocity of the compressor (that blows air on the cathode side). The control variable $u(t) = v_{cm}(t)$ is the compressor motor voltage. $w(t)$ is the disturbance and it represents the stack current, that is $w(t) = I_{st}(t)$. It is assumed in the original model (1) that the hydrogen comes from a pressurized tank. In the follow up of this study, hydrogen will be supplied by a reformer (a fuel processing system) that produces hydrogen from hydrogen rich fuels. The main task in this model is to determine the cathode air molar flow rate, or more precisely the compressor angular velocity that determines the compressor molar flow rate, which is equal to the cathode molar flow rate.

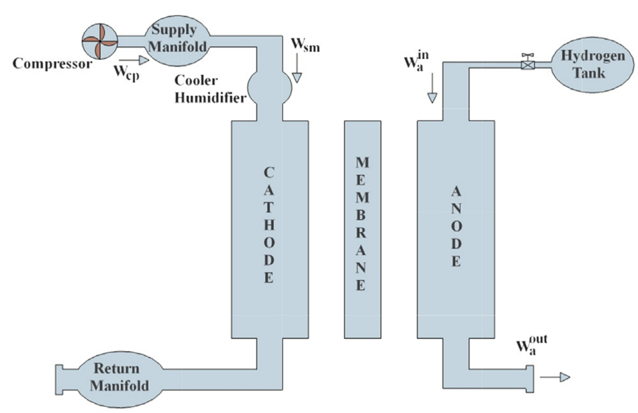

Fig. 1 – PEMFC schematic.