Available online at www.sciencedirect.com
ScienceDirect
Engineering Fracture Mechanics 73 (2006) 2642–2661
Engineering Fracture Mechanics
www.elsevier.com/locate/engfracmech

Determination of mixed mode cohesive laws

Bent F. Sørensen [a,*], Peter Kirkegaard [b]

[a] *Materials Research Department, Risø National Laboratory, Frederiksborgvej 399, P.O. Box 49, DK-4000 Roskilde, Denmark*
[b] *IT-Service Department, Risø National Laboratory, DK-4000 Roskilde, Denmark*

Received 1 November 2005; received in revised form 4 April 2006; accepted 13 April 2006
Available online 22 June 2006

Abstract

A novel approach is proposed for the determination of mixed mode cohesive laws for large scale crack bridging problems. The approach is based on a plane, two-dimensional analysis utilizing the J integral applied a double cantilever beam specimens loaded with uneven bending moments. The normal and shear stresses of the cohesive laws are obtained from consecutive values of the fracture resistance, the normal and tangential displacements of the end of the cohesive zone. The data analysis involves fitting and determination of partial differentials. This is done by a numerical method using Chebyshev polynomials. The accuracy of the numerical procedure is investigated by the use of synthetic data. It is found that both the shape and peak stress of the cohesive law can be determined with high accuracy, providing that the data possess low noise and a sufficiently high number of datasets are used. The investigation leads to some practical guidelines for experimental use of the proposed approach.

Keywords: Mixed mode crack bridging; J integral; Cohesive laws; Bridging laws

1. Introduction

Cohesive laws, introduced by Dugdale [1] and Barenblatt [2] to describe the mechanical stress-separation behaviour of a failure process zone, are widely used in models of problems involving crack initiation and growth. While cohesive laws represent the entire failure process zone, bridging laws are used for describing crack bridging phenomena behind a crack tip, at which a singular stress field exists. Due to their similarities, we will not make a sharp distinction between bridging laws and cohesive laws in the present paper. A beauty of the bridging/cohesive law approach is that it can be applied to a number of very different problems on various scales. For instance, model studies have given valuable understanding of microscale failure mechanisms, such as the formation of a crack tip plastic zone [3] and the development of fibre-bridged matrix cracks in ceramic matrix composites [4]. Other investigations have used cohesive laws for strength prediction of macroscale specimens, such as the strength of panels with holes or notches and strength of adhesive joints [5–10]. It has been

* Corresponding author. Tel.: +45 4677 5806; fax: +45 4677 5758.
E-mail address: bent.soerensen@risoe.dk (B.F. Sørensen).

0013-7944/$ - see front matter
doi:10.1016/j.engfracmech.2006.04.006

Nomenclature

a_{ij}	Chebyshev coefficients
f	approximate function (product of Chebyshev polynomials)
i	integer subscript
j	integer subscript (coordinate axis; polynomial degree)
k	integer subscript (polynomial degree)
n	shape parameter of power law function
n_i	normal vector
p	integer
r^2	goodness of fit
u_i	displacement field (vector)
x_i	Cartesian coordinate system
$\bar{x}$	transformed normal end opening displacement
$\bar{y}$	transformed tangential end opening displacement
B	specimen width
E	Young's modulus
H	specimen height
L	bridging zone length
J	J integral value
J_0	crack tip fracture energy
J_{ss}	steady-state fracture resistance
J_{tip}	J integral value evaluated around crack tip
M	number of data points per phase angle
M_1	moment applied to one arm of DCB specimens
M_2	moment applied to the other arm of DCB specimens
N	number of data points in the group of datasets (all phase angles)
S	curve length
T_i	Chebyshev polynomial of the first kind of degree i
W	strain energy density
δ	crack opening displacement
δ_0	end opening where cohesive stresses vanish
δ^*	end opening displacement
δ_{n}	normal displacement
δ_{t}	tangential displacement
δ_{n}^*	normal displacement of end opening
δ_{t}^*	tangential displacement of end opening
φ	phase angle of opening
ε_{ij}	strain tensor
ν	Poisson's ratio
ω	noise parameter
σ_{ij}	stress tensor
ξ	parameter of power law function
σ_{n}	cohesive normal stress
σ_{t}	cohesive shear stress
$\hat{\sigma}_{\mathrm{n}}$	peak normal stress
$\Delta\varphi$	difference between phase angles
Δ^2	sum of squared deviations
Γ	integration path for J integral
Γ_{ext}	integration path along external boundaries of specimen