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A weight function methodology for the assessment of embedded and surface irregular plane cracks

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Abstract

A low cost numerical tool for the calculation of mode I stress intensity factors K in embedded and surface irregular cracks is presented in this paper. The proposed tool is an extension of the O-integral algorithm due to Oore and Burns for the assessment of embedded plane cracks using the weight function methodology. The performance of the O-integral is assessed first by comparing its K results to exact solutions for embedded elliptical and rectangular cracks. From the analysis of this data it is found that the error in the K results systematically depends on the crack aspect ratio and the local crack front curvature. Based on this evidence a corrective function is derived in order to remediate the limitations of the O-integral. Solutions due to Newman and Raju are used to account for the effects of free surfaces and finite thickness. The accuracy of the proposed procedure is assessed by solving a number of examples and by comparing the obtained results to those available in the literature.

Keywords: Fracture mechanics; Stress intensity factor; Three-dimensional cracks; Numerical algorithms; Surface cracks

1. Introduction

Design standards based on theory and rules of good art allow to minimize the risk of failure of structures of all types. Among those, best known are ASME PVPC Code [1], used by manufacturers and operators of pressure vessels and pipes, and others applicable to specific industries. These standards mostly come from USA (API, SEAL, STUMP, etc.), and from European countries (DIN, ISO, etc.). Another interesting aspect refers to the evaluation of equipment in operation that, due to fabrication errors or accumulated damage in service, show discontinuities or crack-like defects that are not allowed by the manufacturing codes. Nowadays there is an advanced theoretical understanding and an array of experimental and numerical tools to address this engineering problem; such as failure analyses, root cause assessments, testing of laboratory samples, numerical analysis for design and re-rating, etc. A set of documents and recommended practices standardize the analysis

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Nomenclature

a, c	crack dimensions: half depth and half width, respectively
r, r^*	crack aspect ratio
b	half width of a plate
t	plate thickness
K	stress intensity factor
K_{corr}	corrected stress intensity factor
K_I	mode I stress intensity factor
$K_{Q'}$	stress intensity factor at a point Q' on the crack front
σ	opening stress
σ_0	remote uniform stress
σ_Q	opening force intensity (pressure) at a point Q
A	crack surface area
s	crack front
ds	elemental length of the crack front s
$l_{QQ'}$	distance from the load point Q to the point Q' on the crack front
ρ_Q	distance from point Q to the center of the elemental length ds
$W_{QQ'}$	weight function
χ_n	non-dimensional normalized curvature of the crack front
ϕ	parametric angle for the crack geometry
$r(\theta)$, θ	polar coordinates
f_c	corrective function
P, R, G	defined functions of r
f_1, f_2	defined functions of χ_n
p, q, t	coefficients in f_2, depending on the parameter r
Y	configuration function
Y_b	configuration function for the crack in an infinite body
Y_b^C	Y_b solution obtained by using the Calyf program
λ_s	free surface correction factor
f_ϕ	Newman and Raju solution function for an ambedded elliptical crack
Q	parameter approximating the square of the elliptical integral of second type
F_S, H	Newman and Raju auxiliary functions
σ_m, σ_b	remote membrane and pure bending stresses, respectively
t_m, t_b	normalized values of σ_m and σ_b, respectively
f_S	auxiliary function for F_S
g_S, f_w	auxiliary functions for f_S
M^S	auxiliary parameters for f_S
q	parameter related to the deformation of a circumference of radius one

of structures in service, which purpose is to prevent accidents and at the same time to reduce repair costs. The British documents BSI PD 6493 [2], CEGB R6 [3]; and the American documents EPRI (Electric Power Research Institute) of 1981 and 1990 [4,5] were among the first bodies to address the problem, leading to present day procedures such as the British Standard BS 7910 [6], which replaces the PD 6493, the European standard procedure for structural integrity assessment SINTAP [7] and the American Petroleum Institute Recommended Practice API RP 579 [8].

Assessing the engineering integrity and life expectancy of a cracked component or structure, either under service conditions or during the design stage requires the determination of fracture parameters as the stress intensity factor K. In this sense, most of the above-mentioned codes and standard procedures use assessment rules with a certain degree of conservatism. For example, API RP 579 models embedded or surface cracks as