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Simulation of size-effect behaviour through sensitivity analyses

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Abstract

Sensitivity analysis techniques are applied to the simulation of size effect behaviour. The scale factor is included in the discretised equilibrium equations. A gradient-enhanced damage model is used. The sensitivity of the equilibrium path with respect to the loading factor is then obtained through the direct differentiation method. Particular attention is paid to the proper differentiation of constitutive internal variables. The predictive possibilities of the algorithm are illustrated by means of an example.

Keywords: Size effect; Finite element method; Sensitivity; Direct differentiation method; Gradient-enhanced damage model

1. Introduction

Advanced methods of design sensitivity analysis are not only needed in optimisation and reliability problems, but are also a valuable complement to any generic finite element computation. In this context sensitivity is understood as the derivative of any measure of the structural response with respect to a design parameter. Size-effect phenomena are essentially a matter of structural performance vs. a scale factor [1] and provide therefore a preferential environment for the application of design sensitivity algorithms.

Several techniques exist for evaluation of the sensitivity to design parameters [10]. Among these, only the direct differentiation method (DDM) seems to be adequate for solids with material non-linearities. The DDM is essentially based on the application of the implicit function theorem to the non-linear algebraic equations which result from the finite element discretisation. When advanced material models are used the dependence of internal parameters on the design parameters must also be considered. This is of special importance when local loading/unloading conditions depend on global variables, as in gradient-enhanced models.

This paper provides insight in how the size effect behaviour can be studied by means of sensitivity algorithms. In particular, the DDM is applied to a gradient-enhanced damage model for concrete. First, the scale factor is incorporated into the discrete equilibrium equations. Then, the sensitivity of the peak load to

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the scale factor is formulated. The exact computation of the peak load is also commented on. The possibilities of the proposed method are illustrated by means of an example.

2. Size effect in gradient-enhanced quasi-brittle solids

A reference solid Ω in plane stress conditions is considered. A scaled solid $\Omega^{(s)}$ is introduced by means of a factor $s \in \mathbb{R}$,

$$\Omega^{(s)} = \{\mathbf{y} \in \mathbb{R}^2 | \mathbf{y} = s\mathbf{x} \quad \text{with } \mathbf{x} \in \Omega\}. \tag{1}$$

This notation implies that

$$\Omega^{(1)} = \Omega. \tag{2}$$

The behaviour of the scaled solid is, in the absence of body forces, governed by the boundary value problem

$$\begin{aligned} &\nabla \cdot \boldsymbol{\sigma} = \mathbf{0} && \text{in } \Omega^{(s)}, \\ &\mathbf{u} = s\,{}^{\tau}\bar{\mathbf{u}}_r && \text{on } \partial\Omega_1^{(s)}, \\ &\boldsymbol{\sigma} \cdot \mathbf{n} = {}^{\tau}\bar{\boldsymbol{\sigma}} && \text{on } \partial\Omega_2^{(s)} \end{aligned} \tag{3}$$

at each instant τ, where $\boldsymbol{\sigma}$ is the stress tensor, $\mathbf{u}$ is the displacement field, $\partial\Omega_1^{(s)} \cup \partial\Omega_2^{(s)} = \partial\Omega^{(s)}$, $\mathbf{n}$ is the outward normal vector to $\partial\Omega$, ${}^{\tau}\bar{\mathbf{u}}_r$ are the prescribed boundary displacements in the reference solid and ${}^{\tau}\bar{\boldsymbol{\sigma}}$ is the prescribed boundary loading. In a context of quasi-static loading, τ is a parameter that merely orders the succession of events. If a linear elastic stress–strain relation is considered, together with a linear kinematic relation between the strain and displacement fields, it can easily be demonstrated that the stress field that provides a solution to (3) does not depend on the scaling. In other words, the field

$$\boldsymbol{\sigma} : \Omega^{(s)} \to \mathbb{R}^2 \otimes \mathbb{R}^2 \tag{4}$$

does not depend on s, i.e.,

$$\frac{\partial \boldsymbol{\sigma}}{\partial s} = 0. \tag{5}$$

Additionally, it can be demonstrated that the displacement field

$$\mathbf{u} : \Omega^{(s)} \to \mathbb{R}^2 \tag{6}$$

satisfies the relation

$$\mathbf{u} = s\mathbf{u}_r, \tag{7}$$

where $\mathbf{u}_r$ is the displacement field corresponding to the solution of (3) in the reference solid Ω.

In quasi-brittle materials the stress–strain relation is not linear. If a damage model is considered, this is expressed as

$$\boldsymbol{\sigma} = (1 - \omega)\mathbf{D}\boldsymbol{\varepsilon}, \tag{8}$$

where $\mathbf{D}$ is the elastic constitutive tensor and $\omega \in [0, 1]$ is a damage parameter that is a function of a history parameter κ representing the maximum value reached by a spatially averaged, equivalent measure $\bar{\varepsilon}^{\text{eq}}$ of the deformation tensor. This is formalised by a damage loading function

$$f = \bar{\varepsilon}^{\text{eq}} - \kappa \tag{9}$$