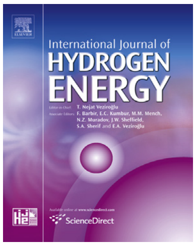
INTERNATIONAL JOURNAL OF HYDROGEN ENERGY 38 (2013) 9043–9051

Available online at www.sciencedirect.com

SciVerse ScienceDirect

journal homepage: www.elsevier.com/locate/he



Characteristics of high Reynolds number flow in a critical nozzle

Junji Nagao [a,*], *Shigeru Matsuo* [b], *Shotaro Suetsugu* [a], *Toshiaki Setoguchi* [c], *Heuy Dong Kim* [d]

[a] *Graduate School of Science and Engineering, Saga University, 1 Honjo-machi, Saga 840-8502, Japan*
[b] *Dept. of Advanced Technology Fusion, Saga University, 1 Honjo-machi, Saga-shi, Saga 840-8502, Japan*
[c] *Institute of Ocean Energy, Saga University, 1 Honjo-machi, Saga-shi, Saga 840-8502, Japan*
[d] *School of Mechanical Engineering, Andong National University, 388 Song Cheon Dong, Andong 760-749, Republic of Korea*

ARTICLE INFO

Article history:
Received 13 December 2012
Received in revised form
12 April 2013
Accepted 30 April 2013
Available online 2 June 2013

Keywords:
Compressible flows
Critical nozzle
Real gas effects
Coefficient of discharge
Simulation

ABSTRACT

A critical nozzle has been used to measure a mass flow rate of gas. It is well known that the coefficient of discharge of the flow in a critical nozzle is a single function of Reynolds number. The purpose of the present study is to investigate the effect of equation of state (EOS) on the coefficient of discharge and thermodynamics properties through the critical nozzle by using H_2 with the help of a CFD method. In computations of the flow field including the stagnation point upstream of the nozzle, the Redlich–Kwong, Lee–Kesler and Peng–Robinson equations of state were employed to take account of real gas effects. As a result of the present computations, coefficients of discharge using the Redlich–Kwong and Lee–Kesler EOS were in good agreement with experimental results in the range of high Reynolds number and the coefficient of discharge decreased with an increase of Reynolds number in the range of $1.0 \times 10^5 < Re < 2.8 \times 10^6$.

1. Introduction

The minimum pressure ratio of the flow to choke is called the critical pressure ratio. Once choked, the flow is no longer dependent on the pressure change in the downstream flow field. In this case, the mass flow rate is determined only by the stagnation conditions upstream of the flow passage. The critical flow meter is defined as a device to measure the mass flow rate with only the nozzle supply conditions, making use of the flow-choking phenomenon at the nozzle throat.

The relationship between the mass flow rate and Reynolds number is as follows:

$$Re = \frac{4\dot{m}_{\mathrm{th}}}{\pi D_{\mathrm{th}}\, \mu_0} \tag{1}$$

where D_{th} is a diameter of the nozzle throat and μ_0 is molecular viscosity at stagnation point. Theoretical mass flow rate at the nozzle throat $\dot{m}_{\mathrm{theo}}$ which is obtained on the assumption of one-dimensional steady isentropic flow, is written in

$$\dot{m}_{\mathrm{theo}} = \frac{A_{\mathrm{th}} p_0}{\sqrt{R_0 T_0}} \left\{ \gamma \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \right\}^{1/2} \tag{2}$$

where p_0, T_0 and R_0 are total pressure, total temperature and gas constant at the stagnation point upstream of the nozzle, respectively. In the practical flow fields, mass flow rate at the

---

[*] *Corresponding author*. Tel.: +81 9086666971; fax: +81 952288587.
E-mail addresses: 11634011@edu.cc.saga-u.ac.jp, matsuo@me.saga-u.ac.jp (J. Nagao).

0360-3199/$ – see front matter Crown Copyright © 2013, Hydrogen Energy Publications, LLC. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijhydene.2013.04.158

Nomenclature

A	cross-section area, m^2
C_d	coefficient of discharge, –
c_p	specific heat at constant pressure, J/kg K
c_v	specific heat at constant volume, J/kg K
D	diameter, m
$\mathbf{E}$, $\mathbf{F}$	inviscid flux vectors, –
E_t	total energy per unit volume, J/m^3
$\mathbf{H_1}$	source term for axisymmetry, –
$\mathbf{H_2}$	source term for turbulence, –
J	jacobian, –
k	turbulent kinetic energy, J/m^3
M	Mach number, –
$\dot{m}$	mass flow rate, kg/s
p	pressure, Pa
p_{b0}	back pressure, Pa
r	radius of the critical nozzle, m
R	gas constant
$\mathbf{R}$, $\mathbf{S}$	viscous flux vectors, –
Re	Reynolds number, –
T	temperature, K
t	time, s
$\mathbf{U}$	conservative vector, –
u, v	velocity components, m/s
v	specific volume, m^3/kg
x, y	Cartesian coordinates, m
z	compressibility factor, –
ε	dissipation rate, J/m^3
γ	ratio of specific heats, –
κ	thermal conductivity, W/m K
μ	molecular viscosity, Pa s
ρ	density, kg/m^3
τ	shear stress, Pa
θ	angle of attack, °

Subscripts

0	stagnation point
b	back pressure
c	critical point
t	turbulence
th	throat
theo	theory

nozzle throat is different from theoretical one (Eq. (2)) because of existence of boundary layer on the wall. The relationship between practical and theoretical mass flow rates is written in

$$C_d = \frac{\dot{m}}{\dot{m}_{theo}} \tag{3}$$

where $\dot{m}$ is the practical mass flow rate at the nozzle throat and C_d is the coefficient of discharge.

From previous researches, many studies have been conducted to predict the mass flow rate through a flow passage because of practical importance in a variety of industrial and engineering fields. Nakao et al. [1] showed that the coefficient of discharge varies significantly with the decrease of Reynolds number in the range of low Reynolds number. After this research, Nakao et al. [2] predicted the discharge coefficient of many kinds of gases in the range of low Reynolds number by the theoretical analysis.

Of many kinds of working gases employed in industrial fields, it has been recognized that hydrogen gas is one of the most promising gases as a future alternative energy source. In such an application, a precise measurement of mass flow rate is of practical importance for mileage and power output of the vehicle. A large number of works [3–6] have been made to investigate the thermophysical properties of hydrogen gas, which are specified by different kinds of the equations of state, the critical back pressure ratio, the compressibility factor of hydrogen gas and so on. Nakao [7] and Morioka et al. [8] have conducted the mass flow rate measurement of hydrogen gas using a critical nozzle and indicated that the coefficient of discharge of hydrogen gas exceeds unity in a specific Reynolds number regime and it decreases with an increase of Reynolds number. Further, they have vaguely inferred that this unreasonable value of the coefficient of discharge would be due to real gas effects or any other measurement errors. However, no detailed explanation has been made for this abnormal discharge coefficient of high-pressure hydrogen gas. Thus only a few researches have been made on the coefficient of discharge of the high-pressure hydrogen gas flow through a critical nozzle because of difficulties of treatment so far.

In recent years, Kim et al. [9] have investigated effects of turbulence model on the critical nozzle flows using a computational fluid dynamics (CFD) method and showed that the standard k–ε model with a standard wall function predicts the coefficient of discharge with good accuracy in the range of low Reynolds number. They have made further investigations to analyze the boundary layer flows through the critical nozzles and showed that the boundary layers at the critical nozzle throat are turbulent, typically expressed by both the law of wall and the law of the wake. Kim et al. [10] investigated the real gas effect of high-pressure (high Reynolds number) hydrogen gas through a critical nozzle by using a CFD method. They used “User-Defined Real Gas Model” in order to introduce the real gas model that is a function of fluent and indicated that the coefficient of discharge in the range of high Reynolds number decreases with an increase of Reynolds number by real gas effect. Nagao et al. [11] investigated the real gas effect on discharge coefficient and thermodynamic properties through a critical nozzle by using working gases of N_2, CH_4 and CO_2 with the help of a CFD method. Further, they described that the coefficient of discharge of real gas for N_2, CH_4 and CO_2 decreases with an increase of Reynolds number, and the compressibility factor and the ratio of specific heats change compared with results of ideal gas in the range of high Reynolds number ($Re = 1.0 \times 10^5 \sim 1.0 \times 10^6$). However, the coefficient of discharge and the details of thermodynamic properties at the nozzle throat have not been clarified in the range over $Re = 1.0 \times 10^6$ satisfactorily.

In these computational studies, the Redlich–Kwong equation of state was used to consider the real gas effect for