Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Engineering Fracture Mechanics 72 (2005) 2758–2773
Engineering Fracture Mechanics
www.elsevier.com/locate/engfracmech

# Effect of crack face contact and friction on Brazilian disk specimens—A finite difference solution

Avraham Dorogoy [1], Leslie Banks-Sills *

*The Dreszer Fracture Mechanics Laboratory, Department of Solid Mechanics, Materials and Systems, The Fleischman Faculty of Engineering, Tel Aviv University, 69978 Ramat Aviv, Israel*

Received 10 January 2005; received in revised form 3 May 2005; accepted 17 May 2005
Available online 18 August 2005

**Abstract**

A full curvilinear transformation is employed to study the effect of contact and friction on Brazilian disk specimens containing a crack and subjected to concentrated loads at angles $0° < \vartheta < 90°$. Homogeneous and bimaterial disks made of glass and epoxy are considered. The effect of loading angle and friction coefficient on the stress intensity factors, as well as the contact length is studied. The results are compared to available semi-analytical and finite elements results. It is found that when the crack faces are in contact without stick zones, an increase in friction causes a decrease of the normal gap, tangential shift and stress intensity factors. When stick conditions appear in the contact zone, an increase in the coefficient of friction also results in increasing the stick zone within the contact zone.


*Keywords:* Brazilian disk specimen; Interface crack; Stress intensity factor; Contact; Friction; Finite difference method

## 1. Introduction

The bimaterial Brazilian disk specimen shown in Fig. 1 has been used in many investigations (see for example [21,22,6,7]). The Brazilian disk specimen is used for determining the critical interface energy release rate $\mathcal{G}_{ic}$ as a function of the phase angle $\psi$. The phase angle depends upon the ratio between the stress intensity factors $K_1$ and $K_2$. The specimen is suitable for a large range of phase angles. For experiments in which the loading angle is greater than some value or the crack is long, depending upon the materials tested, a

* Corresponding author. Tel.: +972 3 640 8132; fax: +972 3 640 7617.
*E-mail addresses:* dorogoy@techunix.technion.ac.il (A. Dorogoy), banks@eng.tau.ac.il (L. Banks-Sills).
[1] Present address: Material Characterizing Laboratory, Faculty of Mechanical Engineering, Technion, 32000 Haifa, Israel.

0013-7944/$ - see front matter 
doi:10.1016/j.engfracmech.2005.05.005

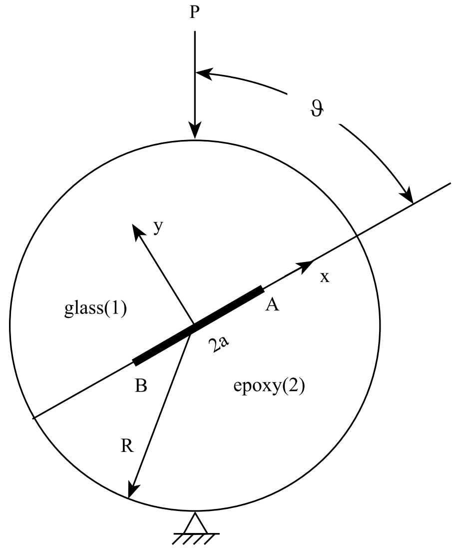


Fig. 1. A Brazilian disk specimen subjected to a concentrated force at loading angle $\vartheta$.

large contact zone occurs between the crack faces at crack tip B. The aim of this investigation is to study the effect of contact and friction on the displacement field and the stress intensity factors of crack tip A.

The elastic two-dimensional problem of contact with friction is solved numerically using the finite difference technique. The two-dimensional equilibrium equations and boundary conditions in an orthogonal curvilinear coordinate system are formulated. The physical domain is mapped by a transformation into a numerical orthogonal domain. The central difference stencil employed may be found in Dorogoy and Banks-Sills [13]. An iterative algorithm which does not require load increments is employed for solving interface fracture problems with contact and friction subjected to a monotonically increasing load. The formulation and the iterative algorithm is given explicitly in Dorogoy and Banks-Sills [13]. It may be noted that for stationary and receding contact, the stresses in the contact area are proportional to the load; while, the extent of contact is independent of it [15,19]. Note that with stationary contact, the crack does not open; for receding contact, it opens partially.

In Section 2, the transformation of the physical domain and its mesh into the numerical region in which the solution is carried out is delineated. The physical domain is characterized by polar coordinates while the numerical region is Cartesian. Two methods for calculating stress intensity factors for an open crack or one which has an infinitesimally small contact zone at the crack tip are employed in this study. In Section 3, the first method in which numerical data is fit to the asymptotic expansion of the tangential shift and normal gap of the crack face is presented. Implementation of the second method, which is a version of the *M*-integral, is described. Two more methods are used for calculating stress intensity factors of crack problems with large contact zones and friction. In the first, numerical data is fit to the asymptotic expansion of the tangential shift. In the second, an extended *J*-integral is used to treat a large contact zone and friction. These last two methods are described in Dorogoy and Banks-Sills [13]. Solution of the problem of a homogeneous disk containing a crack is presented and discussed in Section 4. First, results of the stress intensity factors for the homogeneous disk with an open or closed crack without friction are compared to results presented by Atkinson et al. [5]. Next, the influence of contact and friction on the stress intensity factors, crack face normal gap and tangential shift is presented. The stress intensity factors for open cracks are calculated by applying the *M*-integral and the displacement correlation method. The stress intensity factors for closed cracks with or without friction are calculated by applying a *J*-integral derived for this case