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A multiphysics model for magneto-electro-elastic laminates

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ABSTRACT

An efficient, geometrically nonlinear, multiphysics plate model for analyzing magneto-electro-elastic composite laminates is rigorously developed by applying the variational asymptotic method. By taking advantage of the inherent small parameter characterized by the ratio of the thickness to the in-plane deformation of the plate, we systematically reduced the original multiphysically coupled three-dimensional model to a series of two-dimensional plate models. A companion one-dimensional through-the-thickness analysis provides necessary constitutive models for the plate analysis. For practical uses, we also fit the asymptotically correct second-order electromagnetic enthalpy into a generalized Reissner–Mindlin model. Three-dimensional multiphysical components such as displacement/strain/stress fields as well as the electric/magnetic potentials and fluxes are obtained through a recovery process. Results for sample problems featuring electromagnetic and elastic coupling as well as characteristically different load/boundary conditions and material configurations are compared with exact solutions to systematically investigate the prediction capability of the present model.

1. Introduction

As an analogy with the exhibition of electromechanical coupling by piezoelectric materials, magnetic materials respond to an externally applied magnetic field (H) by exhibiting a shape change which is known as magnetostriction, demonstrating the Joule effect. On the other hand, these magnetic materials also demonstrate the Villari effect indicated by changing their magnetization and consequently the magnetic induction (B) in response to the applied stress. Moreover, for composites containing piezoelectric phases and piezomagnetic phases there exists a magnetoelectric coupling effect. This capability of interactive transformation of magnetic, electric and mechanical energies from one type to another, has received considerable and increasing attentions for developing smart or active structures (Huang, 1998; Nan, 1994; Tanaka et al., 2009; Ueno and Higuchi, 2005). Application of this kind of smart composite materials spans from electronic package materials, magneto-electric-mechanical actuators and transducers, coil-less magnetic force control devices, to nuclear fusion reactor components. The new concept of multi-functional materials/structures featuring elastic, electric, and magnetic interaction is also likely to bring a new dimension to the development of advanced lightweight, multi-functional aerospace structures, among which many critical thin-walled components take the form of beams, plates, shells, and stiffened panels.

The promising application of piezoelectric and piezomagnetic composites makes it imperative to develop new methods and analysis tools for better understanding mechanisms and behaviors of such structures which are subjected to mechanical, electric and magnetic interactions. Recently, increasing researches have been focused on studying static and dynamic behavior of smart plates. Various forms of constitutive equations for magneto-electro-elastic solids were derived based upon a thermodynamics approach using Gibbs free energy function (Pérez-Fernández et al., 2009; Soh and Liu, 2005). Pan et al. developed a series of three-dimensional (3D) exact benchmark solutions to analyze the static behavior of multilayered anisotropic piezoelectric and piezomagnetic composite plates under cylindrical bending and simply supported boundary conditions (Pan, 2001; Pan and Han, 2005; Pan and Heyliger, 2003). These solutions are highly valuable for illustrating the complicated multiphysics nature of the interactive fields, however, they are restricted to a few specific problems with idealized material types, geometry and boundary conditions.

To overcome this limitation, a number of more generalized methods have been developed to explore the 3D behavior of the

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magneto-electro-elastic laminates. Most of these methods are constructed from the layerwise or discrete-layer lamination theory, using various approximations to split the through-thickness behavior and the planar behavior of the laminate into separate functions. In Heyliger and Pan (2004) and Heyliger et al. (2004), an approximate discrete-layer model was developed to investigate the through-thickness variation of the elastic, electric, and magnetic fields of laminates composed of elastic, piezoelectric, and magnetostrictive layers. Semi-analytical approximation solutions to the weak form of the governing equations of equilibrium, charge, and magnetic flux are obtained for infinitely long laminates under cylindrical bending and rectangular laminates with arbitrary edge boundary conditions. By using a state space formulation, Chen and Lee (2003) constructed an alternative solution approach to investigate the nonhomogeneous magnetoelectric plates, where elastic displacements and electric/magnetic potentials as well as the transverse stresses, electric displacement, and magnetic flux are introduced as state variables. Methods based upon the combination of layerwise plate theory with finite element method (FEM) have also been used to analyze the linear static and dynamic behavior of multilayer smart plates. Examples include a layerwise FEM for analyzing piezoelectric composite plates (Saravanos et al., 1997) and a quasi-analytical through-thickness FEM for functionally graded magneto-electric-elastic plates (Bhangale and Ganesan, 2006). In contrast to the classical FEM based on the principle of virtual displacements, layerwise mixed finite element formulation is built on Reissner mixed variational theorem (RMVT), where transverse stress assumptions are made in the framework of RMVT and the resulting finite elements describes *a priori* interlaminar continuous transverse shear and normal stresses. A detailed review on Reissner variational principle can be found in Carrera (2001). Recently, the layerwise mixed finite element formulation has been extended to analyze coupled magneto-electro-elastic problems. Related work has been reported for a partially mixed finite element formulation (Lage et al., 2004) and a layerwise modeling of magneto-electro-elastic plates in Phoenix et al. (2009). In a series of resent researches, by combining a unified Formation framework with the principle of virtual displacement or RMVT, Carrera et al. developed advanced finite element formulations for the multifield problems and multilayered plate structures (Carrera et al., 2008; Carrera and Nali, 2010; Robaldo et al., 2006).

Aimed to avoid the extensive computational cost and modeling complexity associated with 3D or layerwise approaches, various simplified or single-layer based plate models have been developed for smart plates. A simplified plate model based on the third-order shear-deformation theory (TSDT) is developed to model the piezoelectric composite laminates (Zhou, 2001). Mitchell and Reddy (1995) presented a hybrid plate formulation for piezoelectric composite laminates, where an equivalent single-layer TSDT is used for the mechanical displacement field and the electric potential is modeled using a layerwise discretization in the thickness direction. Because plane stress assumption is adopted during the modeling, this model may only suitable for thin plates with some specific load and boundary conditions. In Kant and Shiyekar (2008), an analytic solution is developed for cylindrical bending of a piezoelectric laminate with elastic displacement terms assumed taking the form of TSDT and the electric potential being obtained by solving a second order differential equation. Most recently, based on FSDT, Milazzo et al. have developed both analytic and FEM equivalent single-layer models for static and dynamic analysis of multilayered magneto-electro-elastic (MEE) plates (Alaimo et al., 2013; Milazzo, 2012). By introducing a virtual mass incremental factor, a fluid–structure interaction model has been developed for free vibration analysis of fluid-loaded MEE plates (Chang, 2013). More complicated models have been developed for nonlinear dynamic analysis. By combining the classical plate theory (CPT) with an energy-based statistical magnetomechanical model, Datta et al. (2008) studied the nonlinear dynamical response of an unimorph structure having a magnetostrictive iron-gallium patch to a non-magnetic aluminum substrate. Several underlying assumptions were made on the total energy function as well as kinematic relations. A method which combines CPT with boundary element method was also proposed (Milazzo et al., 2006). Hasanyan et al. (2005) have developed a geometrically nonlinear model for fully coupled magneto-thermo-elastic kinetics of laminated composite plates, with its kinematic relations constructed by the injection of the first-order shear-deformation theory (FSDT) with von-Kámán strain definition. Despite of the successfulness of the aforementioned simplified plate models in analyzing some multi-physically coupled plate problems, these approaches have three major disadvantages: (1) *a priori* assumptions on kinematics and the electromagnetic potentials introduced by these methods are naturally extended from the analysis of isotropic homogeneous elastic problems. And these assumptions may not be justifiable for the multiphysically coupled, heterogeneous, and anisotropic structures. (2) The obtained results are only compared with other similar single-layer based plate models such as CPT and/or FSDT, their accuracy cannot be determined without extensive validations against 3D exact solutions. (3) It is difficult for an analyst to determine which assumption should be chosen for efficient yet accurate analysis of a particular problem.

Recently, based on the variational asymptotic method (VAM), a series of rigorous, asymptotic plate models for heterogeneous and anisotropic functionally graded composite laminates as well as piezoelectric plates are developed (Chen and Yu, 2009; Liao and Yu, 2008, 2009; Zhong et al., 2013). These models have been proven to have excellent compromise between the efficiency and accuracy. It is worth noting that based upon variational principle Altay and Dökmeci also provided a theoretical and rigorous derivation of the fundamental equations for electro-magneto-elastic solids with applications to shells and laminae equations (Altay and Dökmeci, 2010). In the present research, we extend our previous work to construct a geometrically nonlinear plate model for smart composite laminates which are under the interactive actions of magneto-electric-elastic fields.

Taking advantage of the small parameter h/l (with h denoting the thickness of the plate and l denoting the characteristic length of the in-plane deformation), VAM can systematically reduce the 3D magneto-electric-elastic coupled problem into a series of efficient high-fidelity 2D models asymptotically correct to different orders of h/l. In the meantime, recovery information which is necessary for calculating 3D magneto, electrical, and elastic components has also been obtained. The present theory can accommodate arbitrary large deformation and global rotation but is subjected only to the restriction that the strains must be small. No *a priori* assumptions on the kinematics or electric or magnetic variables have been invoked. Continuity conditions for primary variables such as elastic displacements and electric/magnetic potentials as well as for the secondary variables like transverse stresses, electric displacement, and magnetic induction are naturally derived through the variational asymptotic process. The accuracy of the simplified plate model is systematically investigated by analyzing various examples with characteristically different material configurations and load and boundary conditions.

2. Three-dimensional formulation

The dynamic behavior of smart composite laminates is governed by the extended Hamilton principle: